How to Play the Guitar by Ear
(for mathematicians and physicists)

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I. Introduction

For years I have tried to learn to play the guitar by ear. My father could strum along to accompany the singing of popular songs, and that is what I wanted to be able to do. So, about 1967, I bought a Yamaha FG-75 steel-string guitar and enrolled in a Fairfax County Community College, Virginia, course on how to learn to play the guitar.

The course was taught by a degreed music instructor, who was well qualified. He taught the basics of how to read music and play simple songs. He taught the basic triad-based chords. My only criticism of him is that he disdained country/western music, which I enjoyed very much. He said disparagingly that there were basically only six different types of country songs. He was evidently unimpressed with the simplicity of country music. Although the fact that it has simple, catchy melodies, is understandable, and the lyrics are about real life (happiness, tragedy, love, unrequited love, folk songs, ballads) did not impress him, those are the very reasons I enjoyed it and wanted to learn to play the guitar.

I already knew how to read music from secondary school, where I played the trombone. With this background, learning to read music for the guitar was not difficult. The treble clef is used instead of the bass clef, but everything else is about the same (flats, sharps, note durations, key signatures, terminology). The big difference is that the trombone is a "one-note" instrument, whereas on the guitar (like the piano) many notes can be played at the same time. This represents a substantial increase in complexity. Furthermore, unlike the piano, in which each tone is represented by a single key, a particular tone can be played a number of different ways on the guitar (since there are six strings, each covering an octave or more). The fact that several notes may be played simultaneously means that the guitar, like the piano, can play chords (which, by definition, are simply a selection of several different notes played at the same time).

The additional complexity of the guitar means that it takes somewhat longer to master than a "one-note" instrument, but instruction once a week for one semester, plus some daily practice, is sufficient time to accomplish the basics. At the end of the course, I could play Valse Lente – a beautiful slow waltz with lots of pretty chords -- from the music. I could play a number of other songs, such as Malagueña, Marty Robbins' El Paso, and Wheels, all from music. I could not play a new song by sight, but could master it with some work.

At this point, I knew how to play the guitar, but my knowledge was useless with respect to accomplishing what I wanted to do – accompany popular songs in any key by ear. Learning each new song required a lot of hard work and memory, and changing keys was out of the question. Since I soon realized that there was no way my then-knowledge of the guitar could help me realize my goal, I lost interest in the guitar. I would pick it up every few years, relearn Valse Lente or Malagueña or Wheels, and drop it again.

On a business trip to the Philippines, I came across some fine-sounding locally made guitars for about $100 (the Yamaha had cost $75 new), and I bought one. I liked playing it much more than the steel-string model that I had originally bought. It was a "classical" style guitar, with nylon strings instead of steel strings. Another difference between this one and the "country" style Yamaha that I had started with is that the neck is wider.

These two differences make the classical (or “folk”) guitar substantially easier to play. The wider neck makes it easier to form the chords, particularly for someone with large hands. The nylon strings are easier to hold down than the steel strings, and your fingers do not get sore, as they do with steel strings. Also, it is easier to play “barre” chords (in which your index finger presses some or all of the strings at a fret). The only problem for some people, especially those who like country/western music, is that nylon strings do not produce the same (traditional) “sound” as metal strings.

About 15 years later, I took another “first course” in guitar, at Pima County Community College in Tucson, Arizona, but it was a repeat of the Fairfax County course, and brought me no closer to my desired goal. Then, in a Tucson restaurant one evening, I heard a superb guitar player, Jorge Lopez, who played exquisitely. He played in the classical style in which the thumb plays (picks) bass notes and the fingers play treble notes. He played mostly Mexican and Spanish pieces, which I liked very much. I asked whether he gave lessons, and he said that he did. I began lessons with Jorge.

Jorge did not use standard staff (stave) music notation, but “tablature” notation, in which each string to be pressed is indicated on a six-line staff (in which each line of the staff represents a string). After a while, I could play some beautiful Mexican and Spanish tunes. Jorge would write down the tablature representation of any song I requested, so I learned a number of other favorites as well. Once again, however, I could play only the version of the song that I read from the music, and changing keys was impossible. Memorizing songs was difficult and slow, so I was essentially “chained” to the sheet music. Despite much additional work and the ability to play many more pretty tunes, I was no closer to my goal of accompanying songs, and after some time I again lost interest. I produced a record (“Conquistador”) of Jorge’s favorite songs. Jorge moved to Toronto, where I imagine he still lives (and entertains). I was no closer to my goal of learning to accompany popular songs in any key by ear.

While I was living in Tucson, a group of friends started a small band at noontime, and I learned to play the baritone horn (similar to the euphonium). I had always thought that the baritone had a more mellow sound than the trombone, and in band arrangements it seemed to play the melody much more than the trombone did. I enjoyed that very much. We played a lot of marches by John Phillip Sousa and others. I then moved to Sierra Vista, where I joined the Cochise County Community College band. We played a wide variety of music, and that was extremely gratifying. Things changed, I moved from Sierra Vista, and the amount of music in my life dropped to near zero. Even my spare time while driving the car I spent listening to French and Spanish language tapes, rather than music on the radio or tape player.

My father was very musically talented. He could play the guitar (acoustic, electric, or “Hawaiian steel-string”), piano, organ, electric keyboard, accordion, violin, and bagpipes. He could play a “one-man-band” setup of a drum, harmonica, and guitar (simultaneously). He could play an ocarina, flute, recorder, a jaw harp, and a kazoo. He could even play a “saw.” He could play from music or by ear. He was a “caller” for square dancing. He enjoyed electronics, and built an electronic organ in the 1950s, years before they became commercial items. He built and played a marimba. Although not a perfectionist, he was good at music, and was often asked to lead local parades playing the bagpipes. He played in local groups, such as the Spartanburg “Rhinelanders” German-music band (accordion), and often played to entertain people in “old folks” homes (usually with the accordion or electronic keyboard).

Because he could play by ear so easily, and with so many instruments, I always assumed that he had some sort of natural "ear" for music. He always "knew" what chord to play next in a song. And not just chords -- he could play tunes (melody and harmony) on the piano or accordion by ear, without ever having seen the music. I could never do this, even with the "one-note" instruments that I had played regularly for years (trombone, baritone). With respect to chords (on the guitar), I could try several chords and guess which one sounded best, but I never had the foggiest idea of what chord to play next in a song. There is no way I could play a new piece by ear, on any instrument. I could play a song in chords if the chords were specified, but had considerable difficulty in figuring out which chord to play by myself. Dad showed me an article about the "circle of fifths," and that it was a guide to determining the "progression" of chords in a song. This information was of limited assistance in determining which chord to play next in a song. It narrowed down the possibilities, but that was about all.

After having taken several courses in the guitar and still having no idea how to play a piece or accompany a song with chords by ear, I assumed that I had a "tin ear" that prevented me from playing by ear, even though I could carry a tune singing. I was chained to sheet music. I enjoyed playing in a band, and could learn to play pieces solo, but I resigned myself to the belief that I did not have the ability to play music by ear. I had mastered two instruments (trombone, baritone) and learned to play a few others (harmonica, bagpipe chanter, mandolin, electric keyboard). I had invested a lot of hours in practicing the guitar over the years, and learned two different methods of reading music (standard music notation and tablature). I had learned all the basic chords (major, minor, seventh, in "home" position and barred). In addition to the courses that I took on the guitar, I had bought a number of books on how to learn to play the guitar and on the theory of music. None of these sources gave any insight in how to play by ear. The fact that I never saw courses advertised on how to play music by ear supported my conclusion that this was a natural talent that I did not possess.

The main reason why I concluded that I could not learn to play by ear was that I could not sense what the next chord in a piece was. There were other reasons as well. In grade school, when the choir director asked the choir to sing a particular note (e.g., "Sing a middle C"), I never had any idea what tone to sing. In 1987, a friend of mine asked me to join a "barbershop-quartet" singing group. Since I enjoy music very much, I agreed. It was a disaster. Not only could I not intone specified notes identified by letter (e.g., "Sing a middle C," as the choir director had years earlier requested), but I could not follow the sheet music even when I had heard the correct starting note. In other words, I could not sing written notes of a specified scale, given the first one – I could not sing from musical notation, as I could using a musical instrument. I had thought that I could, since I never had any problem singing the melody of hymns (even unfamiliar ones) in church by following the music. Evidently, however, I was being guided by the piano or organ. In the barbershop group, however, you are "harmonizing" – you are not singing the melody, but a sequence of seemingly unrelated notes that are in harmony with the seemingly unrelated notes that the other members of the group are singing.) Although I have heard a "middle C" hundreds of times, I have no memory of it and cannot reproduce it without hearing it again. In elementary school, I had learned to sing a major scale using the Italian note names (Do-Re-Mi etc.), so I did have a sense of how to spread eight notes over an octave (from one sound frequency to double that frequency). And I can "carry a tune" – sing melodies of songs from arbitrary starting points (keys).

So, based on my inability to reproduce middle C from memory, my inability to sing arbitrarily designated notes on a scale given the base (key) note, and my inability to play by ear with instruments that I had mastered, I concluded that I was simply unable to learn to play the guitar by ear. My "ear" was evidently not sufficiently well tuned to enable me to do this. I had no doubts about my intellectual abilities to learn music, or to learn anything. I earned a PhD in mathematical statistics from the world's leading university in the subject, and I have self-taught myself many things, from auto repair to building computers to systems and software engineering. I have participated in many physical activities (tennis, golf, fencing, archery, judo, jiu-jitsu, Tae Kwon Do, SCUBA diving, cross-country running, skiing), and am in good health (except for one deaf ear in recent years). The fact that I am in good health and have been able to master about anything I wanted made it difficult for me to accept that I could not learn to play by ear, when others could. Perhaps that is why I kept trying, from time to time, for more than three decades.

Another fifteen years passed. I found myself in Botswana, on a two-year consulting contract. As luck would have it, at about the same time that my wife and I arrived, another man – Gordon Atkinson – and his wife arrived from Canada, also on a long-term contract. He was an accomplished guitar player and singer. He could strum chords to accompany any song, by ear, and in any key. Moreover, he played all the country/western, folk, and popular songs that I liked. Gordon said that there was no special secret to it, and that with a little practice I could learn to do it also. This was exactly what I was looking for! I would try one more time!

We became good friends, and in fact rented homes next door to each other. He was quite willing to show me his method of playing the guitar, and I resolved to learn it before my assignment was over. We generally practiced for a couple of hours one night a week, for several months. Over the Christmas season, I spent about an hour a day practicing. I can now strum chords to accompany many songs by ear (despite the fact that I am stone deaf in one ear). It is not simple, and I am not at all perfect at it yet, but it is quite possible to accomplish this with a modest investment in practice. This article will tell you how I did it.

(My learning to play the guitar by ear, and my writing of this article, are a direct result of the "Year 2000 Problem" (the inability of some computer systems to correctly recognize the millennium date change). I was Director of Management Systems for the Bank of Botswana (Botswana's central bank), and had to stay in Botswana over the millennium date-change period (December 1999 - January 2000) to address any date-change problems that might arise. In mid-December my wife left for Christmas vacation in the US for six weeks, and I had a lot of spare time. Before she left, I could not play a single tune by ear in an arbitrary key. As she departed, I told her that my goal was to be able to play ten songs by ear in a number of keys. When she returned six weeks later, I could play virtually any popular song in a variety of keys, and had written this article.)

This method involves only the playing of chords to accompany songs. I still have no idea how my father or others can play the melody as well as the harmony of songs by ear. So far as I am aware, that is a natural talent. It is possible that there is a secret to that, also, but I do not know it. As I mentioned, despite many years of playing two instruments, I have no idea how that is done.

One thing that always bothered me in the books on music theory (at least the books that I found and bought) is that they did not explain at all the “why” of the theory. They described chords (major chords, minor chords, seventh chords, augmented chords, diminished chords, and others) and the circle of fifths, but they offered no explanation as to why certain tones sound good together, while others did not, why a particular selection of chords is used in a song, and why some chords seem to naturally follow others. They did not explain why attention centers on major scales and minor scales, or why musical notation is so complicated (with flats, sharps and keys), or why particular keys are used, or why the basic chords involve just three notes. I have not been exposed to the academic field of the theory of music, and perhaps the problem was that I was buying music theory books in popular-music stores. Perhaps there are books on the theory of music (in universities) that do explain the answers to the many questions that I had, but they are definitely not in the popular press.

I like to understand the reason behind what I do, and just accepting the rote pronouncements of the music theory that I read was very unsatisfying. Recently, I spent some time figuring out the “whys” of music theory. There are very good reasons for the tenets of music theory, and it is very satisfying to know why things are the way they are. Before explaining how to accompany songs with chords, I will explain this theory. I have not seen this theory anywhere else, but I have not researched the topic and this material is probably available from somewhere (but not in music stores!). (I do not have ready access to specialized reference materials here in Botswana, so I have not checked this out.) The explanation is not difficult to someone who has some basic knowledge of physics (sound, wave motion, harmonics) and mathematics (the exponential and logarithmic functions). It is because of this mathematical/physical description of music theory that I refer to this method as one directed to mathematicians and physicists. If you have no interest in understanding music theory, you can still learn to play the guitar by ear – you will just not understand why it works.

II. What to Look for in a Guitar

The reason why I bought a Yamaha guitar was the good reputation of the firm. I had been aware that Yamaha made some of the finest pianos in the world, and I assumed that they made good guitars. They certainly make good baritone horns.

There are lots of good guitars on the market. Listed below are basic features that you should look for or consider in selecting an acoustic guitar. I don’t know much about hardbody (solid electric) guitars, and have no comments on them. You do not need to spend a lot of money for a good guitar. I have bought several guitars that sounded great and played well, and never spent over $150 for one (although that was some time ago – I bought the Yamaha in 1967). If you buy a used guitar, check carefully for cracks in the wood or separations in the joints, or strange vibrations that might suggest these. Nicks, dents, and scratches all affect the sound quality, so do not buy one in poor physical condition unless the sound is good and the price is good. The age of a guitar doesn’t matter that much – they do not wear out like cars.

There are good buys available in private sales, because a lot of people try to learn and then give up. Ask why the seller is selling the guitar. If in doubt, have a local music shop that does guitar repairs check out a used guitar, if you are buying it from a stranger and going to spend much on it. The main things are that it sounds good (resonant, not “dead”), has a “long-lasting” sound (i.e., the tone diminishes slowly after a string is plucked), can be tuned, and is easy to play (strings level and close to frets, but not so close as to "buzz"). Go to a musical instrument store that has a lot of guitars, and strum several, ranging from very expensive to inexpensive. You will find that you do not have to pay a lot to get one that sounds about as good as an expensive one. If you can't find a reasonably priced new guitar, check the newspaper want ads for a used one.

The list of what to look for in a guitar is as follows. For pictures, definitions, and more information, buy any book on guitar basics.

1. The fretboard (fingerboard) is made of very hard, nonwarping wood, such as ebony, rosewood, or walnut.
2. The soundboard is a “springy” wood, such as spruce.
3. The front and back of the guitar have many braces (interior wooden strips), fanning out across the (inside) surface. The braces strengthen the guitar, and they affect its tone. They may be examined with a small mirror, or by removing the strings and inserting your hand in the soundhole. They should not be loose or missing. They should be arranged in a symmetrical pattern, and neatly glued.
4. The grain of the front and back is fine near the middle of the guitar and wider toward the outer edges.
5. The frets are very hard nickel-brass alloy, with so much nickel that they are almost silver colored, not yellow, brassy colored.
6. The strings should be close to the frets, so that you do not have to press the strings down very far.
7. The frets are level, i.e., some are not noticeably lower or higher than others (or else the strings will “buzz” when certain frets are used. The distance between the frets and the strings increases as you approach the body of the guitar. The reason for this is obvious – when you press a string against one fret, the string should not vibrate against any other frets (closer to the body).
8. The neck is straight. On steel-string guitars there is a metal “adusting” rod (truss rod) through the length of the neck, which may be tightened to remove curvature from the neck (i.e., raise or lower the neck slightly, in case, over time, the neck warps slightly and the strings start to buzz the frets). There is an adjusting nut at one end or the other (at the tuning-key end or the body end), that can be turned with a hex (Allen) wrench or other wrench. If the adjustment is at the turning-key end, the adjustment nut is under a small cover held on by one or more screws. If the adjustment nut is on the inside of the guitar, it can be reached directly through the soundhole. Classical guitars use nylon strings, not steel stings, and do not require a truss rod (since the tension on the strings is much less).
9. The tuning machines (post, gear, and tuning key), or “machine heads” on the headstock (used to tighten the strings) should be of very hard metal (nickel alloy or chrome steel), rather than brass. If the guitar is a classical-style one, the string adjusting posts lie flush in the neck. If the guitar is a steel-string one, the string adjusting posts lie perpendicular to (i.e., project from) the neck.
10. If the guitar is a steel-string one, it is usually played with a pick (plectrum). To prevent damage to the guitar front (from the pick), a plastic shield is placed just below the soundhole. If you wish to play with a pick, buy several thicknesses of picks. The thin ones are easier on the strings. My dad used a thumb pick (a curved pick that fits over the first joint of the thumb), but I never saw the advantage in it. Perhaps it is a good idea if you are play steel strings for long

periods, and strum with your thumb. It does produce a much louder sound on the bass notes than does the thumb. You may also use finger picks (that fit over the finger). The thumb and finger picks produce a somewhat different sound from the ordinary (flat, heart-shaped) picks. For rapid strumming, the ordinary pick is the simplest.

11. Acoustic guitars have hollow bodies, and soundholes to allow the air to freely flow in and out of the guitar when the strings are plucked, to achieve the loudest sound possible. (The sound comes mainly from the vibrating soundboard, not from the soundhole(s), and very little directly from the strings. Without soundholes, the soundboard could not vibrate as freely, since more energy would be lost in compressing the air in the guitar than in pushing it in and out of the soundhole(s), and the guitar would sound "dead".) Acoustic guitars usually have flat fronts (soundboards) and backs. These guitars are called "flat-tops." Steel-string models are often shaped like violins, with "landau" or italic-f design cuts for the soundholes instead of a round soundhole. These guitars are called "arch-tops." In general, the larger the guitar body, the louder the sound.
12. The guitar is shellacked (lacquer), not varnished (which would penetrate the wood and affect the sound).
13. If you wish to play standing up, there should be a post to which to attach the neck strap on the bottom of the guitar. If you wish to attach the strap to the place where the neck is attached to the body of the guitar, there should be a post there. Otherwise, you will tie the strap at the end of the neck, just past the last fret.
14. Buy a tuning fork (E or G or D or B will tune open strings; A440 forks are easier to find) to tune your guitar. (You need just one tuning fork – you tune one string to it, and tune the other strings to that string. A440 is the A at the 5th fret on the first (narrowest-gauge) string.) Pitch pipes are not reliable. Electronic tuners are great – fast and accurate – but they are not inexpensive.
15. Tune the guitar by tuning a single string with a tuning fork (or electric tuner), and then tune the other strings relative to that string (by matching the tone of the next higher string to the tone from the same note played using a fret on the next lower string). Check all the strings with an electronic tuner. The strings should all be in tune. Check the harmonic relationships of certain strings, if you know how to do that. (There are several methods of tuning a guitar: relative tuning, tuning according to E notes, tuning with harmonics, tuning to a central string. This article will not describe these different methods.) On solidbody electric guitars, the intonation of each string may be adjusted individually at the bridge saddle. Arch-top guitars may have a movable bridge and slanted saddle to permit adjustment of the intonation. The problem that this adjustment is designed to fix is when a fretted note on a string is not in tune with the second harmonic of the unfretted string. If the fretted note is flat compared to the harmonic, then the string is too long, and its length should be decreased by moving the bridge forward. On acoustic guitars, the bridge and saddle are fixed, and the saddle slanted. You are pretty much stuck with the intonation that is "designed into" the guitar, but changing the string gauges may help. It is difficult to tune light-gauge strings. The easiest-to-tune steel strings are of gauges .012 (inches), .016, .024 (wound), .032 (wound), .042 (wound), and .054 (wound). Normal-gauge nylon strings are .028, .032, .040, .030 (wound), .034 (wound), .042 (wound).
16. The guitar should sound great – a loud, clear, mellow tone that lasts for a long time. No discordant sounds, which may suggest a crack or joint separation. (It is assumed that good-quality strings are on the guitar.)

17. I have been told not to put steel strings on a classical-style (folk) guitar – that it may rip the bridge (where the strings are attached to the soundboard) out of the front of the guitar. The reason for this is that steel strings place much more tension on the bridge, and so it is anchored more firmly to the guitar front on a steel-string model. Willie Nelson does exactly this (puts steel strings on a classical guitar). Steel-string guitars are usually played with a pick, however, and classical (folk) guitars are usually played with the fingers. Without a "pick shield," the front of your guitar will gradually become damaged (steel-string acoustic guitars have pick shields because they are usually played with a pick).
18. Check to see that all joints are solid (neck against body, sides meeting front and back).
19. It is convenient to have marks ("dots" or more elaborate inlaid designs, often of mother-of-pearl or ivory inlay) at certain positions along the fretboard (on the top of the fretboard on on its side) at certain key frets, such as the seventh and twelfth, or the third, fifth, seventh, ninth, twelfth and fifteenth. On a classical guitar, the twelfth fret is usually located where the neck meets the body. The twelve frets cover a full octave. On a steel-string (acoustic) guitar, the neck usually meets the body at the fifteenth fret. On a solidbody guitar, the neck may contain over twenty usable (reachable, playable) frets – almost two full octaves (24 frets). (It is much easier to play a long-necked guitar than a short-necked one. A short solid-body guitar may seem like a great idea for travelling, since it would fit inside a regular suitcase, but it is very difficult to play, except perhaps for a child.) On some guitars, part of the body is cut out below the fretboard out so that you can have a larger body (relative to the length of the neck) and reach more higher-pitch frets. (There is no need for cutouts on solidbody electric guitars, because the sound is captured by the pickups, and the sound quality has nothing to do with the size of the body (it is affected by the quality of the pickups). The body need be only large enough to hold the pickups and controls. Because the body may be smaller, the neck may be much longer, and hold many more frets. The quality of tones at high frets is not very good on a classical guitar, but it is quite satisfactory on an electric guitar. If you traveling a lot, an electric guitar with a small portable amplifier (e.g., a Pignose amp) is handy, because the body can be very small so that the guitar will fit in a suitcase.)
20. If you want to amplify the sound of the acoustic guitar through speakers, purchase an electrified ("electric") acoustic guitar. The incorporation of a pickup in the guitar when it is built does not detract from the quality of the sound. The alternative of attaching a pickup to a nonelectrified acoustic guitar is not as good. The advantage of an electrified acoustic guitar over an electric solidbody guitar is that you can play to yourself or a small group without amplification (no electricity or speakers). Without electrification, a solidbody electric guitar produces hardly any sound at all.
21. Rounded-body guitars look neat, but they are hard to hold.
22. Buy a classical-style guitar if you wish to play classical music or any other kind of music. Do not buy a steel-string model if you wish to play classical music – not only would it not sound right (metallic rather than mellow), but it is hard to play "barre" chords on a steel-string model, even with lightweight steel strings. Buy a steel-string guitar (narrow neck) if you intend to play only country/western music, and want the traditional steel-string sound. In a way, it is easier to strum with a pick on a steel-string guitar, because the strings are much tauter and closer together. Because the strings are a little closer together, however, it is a little harder to select particular strings with the pick.

23. Buy a hard-shell case if you will be carrying your guitar many places, a soft (vinyl or leather) case only if you will be keeping it home. Be careful not to knock the guitar. Each little scratch or dent changes the sound quality, usually for the worse.
24. If you will be traveling by airplane, so that you will have to check your guitar as luggage, buy a heavy-duty case (which looks more like a trunk than a hardshell guitar case).
25. If you buy or are given a guitar in poor physical condition and wish to restore or refinish it, remember to use shellac, not varnish. A water drop placed on a dent will raise it, unless the wood is torn.

III. Some Theory of Music

A. The Issue of How Many Tones and Notes to an Octave

1. Some Definitions, and The "Evenly Tempered" Scale

The first issue to address is why the musical scale (in Western music) consists of twelve tones (seven designated as "natural" notes plus five designated as "accidental" notes), and how those tones are defined (in terms of frequencies). First it is useful to define some terms. A "tone" is the sound produced by vibrating a medium (air, or a wire, or a bar, or a skin) at a particular frequency. A "note" is the name given to a tone. The terms "tone" and "note" are often used interchangeably. An "octave" is a range of tones from a specified tone to the tone of double that frequency. The word "octave" is derived from the Latin word, octo, meaning "eight," and it was introduced when the frequency range from one frequency to its double was divided into eight tones (i.e., eight tones of that range were used as the basis for melodies). The word octave now refers to the frequency range from one frequency to its double, however, no matter how many tones that range is divided into.

At this point, I must point out that the definition of the word "tone" is different in physics and in music. It is also different in different countries. In this article, I will use the physical definition (which also corresponds to the lay concept). In physics, a "tone" is a sound of a specified frequency, such as the sound of the note A, which is a frequency of 440 vibrations per second (or 440 cycles per second, or 440 hertz, or 440 hz). In music, a (physical) tone (sound of a particular frequency) is referred to as a "note," and the frequency of the note is referred to as the "pitch" of the note. A "semitone" (half tone) is the difference, or length of the interval, between two adjacent tones on the 12-tone chromatic scale (used in modern music, and to be define later); e.g., the difference between A and A#, or between B and C, is a semitone. (The scale is AA#BCC#DD#EFF#GG#A, with ascending letters representing higher frequencies.) A "tone" (in music) is the <u>difference</u> between next-adjacent notes, i.e., two notes with a note in between); e.g., the difference between A and B, or between B and C#, is a tone (or "full" tone).

By using the physical (and lay) definition of "tone," I will say that the interval between two tones (sounds of two different pure frequencies) is so many hertz, or so many tones, or so many steps. For example, on the modern 12-tone musical scale, I will say that the difference (or length of the interval) between A and A# (adjacent tones) is one tone or one step, not a semitone or half-tone or half-step. Similarly, I will say that the difference between A and B is two tones (not a full tone or full step). It is understandable that music and physics (and common parlance) would not use the same term for such a fundamental concept as a pure-frequency sound, but it is unfortunate that they use the same word (tone) for quite different concepts (a sound of a given frequency vs. the interval between to sounds of different frequencies). In any event, no confusion should arise as long as this distinction is recognized, and it is known which definition I am using.

A "scale" is a specified set of tones over an octave (just as the twelve inches mark a one-foot ruler). Although there are twelve tones per octave in Western music, most Western musical pieces involve only a selection of eight of these tones. (I will use Western with a capital initial letter to refer to music derived from Greek civilization, and western with a lower-case initial letter to refer to country/western music of North America.) Most people know that an octave is a range of frequencies from a specified frequency to the frequency that is double that. But why has this range been divided into twelve tones (i.e., twelve tones have been selected as a basis for music), and how are the tones specified, and why are most musical pieces based on a subset of eight tones, and how are these tones selected?

The names of the twelve tones used in the Western music scale are A A# B C C# D D# E F F# G G# or A Bf B C Df D Ef E F Gf G Af. (I shall use the symbol # for sharp, as is standard, and the symbol f for flat, which is not.) These tone names are called notes. Tones an integral number of octaves apart have the same note names. Hence a particular note name refers to many different tones (e.g., A can refer to A110, A220, A440, A880, etc.). As mentioned, most Western musical pieces use are based on a particular subset of seven of these twelve tones. The particular selection of notes used for a song is called the "key." It is also called a "scale." For a piece written in the key of C, the notes that are used are A B C D E F G. For a piece written in the key of F, the notes are F G A Bflat C D E. I will say more about keys later on, and why most music is based on a scale of just eight notes. The scale of all twelve notes is called the "chromatic" scale. The scales of the subsets used as the basis for musical pieces (called "diatonic" scales) have various names, such a C, Aminor, Gmajor, and the like (we shall say more about the subset scale names later). The modifier "chromatic," which relates to color, is used to refer to the complete scale since it contains a full range (or spectrum) and rich variety of tones. Since sound, like electromagnetic radiation, is produced by wave motion, the terms from wave theory (wavelength, frequency, octave, harmonic, spectrum) apply.

The fact that most (Western) musical pieces involve only eight tones over an octave may trace its origin to the simple (traditional) flute. Since the human hand has eight fingers opposed to the two thumbs, it is possible to cover up to eight tone-holes on a flute. This means that a simple flute (without a back hole) can play up to nine tones. (If you have played a flute, you know that by blowing harder you get a tone an octave higher, for each tone hole. Hence a simple flute with n holes can actually produce 2(n+1) tones, not just n+1 tones.) If the holes are equally spaced and appropriately distanced from the lip hole (or slot over which the sound vibration is created) so that the tones are evenly spread over a full octave (i.e., from a frequency to its double), then a wide range of melodies may be played. Melodies sound "complete" if they end on the same note as they began (or an octave higher or lower). Hence it is convenient to have the tone holes cover a complete octave, and maybe even an extra note. These considerations would suggest that an octave have at most eight or nine tones. For players of harps, it would be convenient to cover a full octave (from one frequency to double that) with eight fingers. Whatever the reason, the melodies of Western music evolved with most songs based on eight tones to the octave.

The tone holes (holes covered by the fingers) on a simple (whistle-type) flute are equally spaced. (On modern flutes, recorders, or tin whistles, they are not equally spaced, in order to generate a "major" scale – more will be said about this later.) From physics, it can be shown that the frequencies generated by equally spaced holes are equidistant on a logarithmic scale. The logarithm of the frequency associated with a tone hole is inversely proportional to the distance of the tone hole from the lip hole (hole near the lips over which sound passes to generate the vibration). That is, if d1 and d2 are the distances of two finger holes from the slot at the top of the flute (near the lips), then the frequencies f1 and f2 of corresponding to these two holes are related by the equations

$$f1 = a \exp(-bd1)$$
$$f2 = a \exp(-bd2)$$

or $f1/f2 = \exp(b(d2-d1))$,

where a and b are constants (that depend on the size and geometry of the flute), and exp(.) denotes the exponential function. The negative sign occurs because the longer the distance of the hole from the slot, the lower the tone (frequency). If the distances di are expressed as ranging from one to two, then the constant b is in fact ln2 (where ln(.) denotes the natural logarithmic function), so the relationship may be expressed as:

$$f1/f2 = \exp((d2-d1)\ln 2) = 2^{(d2-d1)} .$$

So, the ratio of the frequency (f1) of the highest note (d1=1) to that (f2) of the lowest note (d2=2) is

$$f1/f2= 2^{(2-1)} = 2 .$$

With the notes separated equally (physical distance of holes, or equal intervals on a logarithmic scale), the scale has a "balanced" character, and is called an "evenly tempered" scale. With a total of eight different tones spanning an octave, it is possible to construct a tremendous variety of melodies. (The reason why tones that are equidistant on a logarithmic scale sound physiologically to be an equal distance apart has to do with the design of the ear.)

2. More About the Evenly Tempered Scale

Above, it was stated that a flute's having equally spaced holes produces tones on an "evenly tempered" scale, and it was stated that with this type of scale the notes are "evenly spaced" in a physiological sense. Let's explore the concept of an "evenly tempered" scale a little deeper. An "evenly tempered scale" is defined as one in which the ratios of the frequencies of successive tones in the scale are all equal. That is, if f_i denotes the frequency of tone i and f_{i+1} denotes the frequency of tone i+1, then

$f_{i+1}/f_i = k$,

where k is a constant. For an evenly tempered scale of 12 tones, the ratio of tones that are an octave apart (i.e., the frequency of the higher tone is twice the frequency of the lower tone) is hence

$f_{i+13}/f_i = 2$.

But since $f_{i+1}/f_i = k$ for all I,

$f_{i+13}/f_i = (f_{i+13}/f_{i+12})(f_{i+12}/f_{i+11})\ldots(f_2/f_1) = k^{12}$.

So $k^{12} = 2$, or $k = 2^{1/12}$.

So, the ratio of the frequencies of successive tones on an evenly tempered scale of 12 tones is the twelfth root of 2, or 1.059463094.

The fact that the ratio of successive tones of an evenly tempered scale is constant means that the difference in the logarithms of successive frequencies is also constant, i.e.,

$\ln(f_{i+1}/f_i) = \ln f_{i+1} - \ln f_i = \ln k = k'$,

i.e., the logarithms of successive frequencies of an evenly tempered scale are linearly (equally) spaced.

The "chromatic" scale of Western music is an evenly tempered scale. Why are we interested in evenly tempered scales? One reason is that the tones sound evenly spaced in a physiological sense. They are natural and pleasing to hear. The very important result of this property is that if you sing a song at different pitches (e.g., in different keys or octaves), the melody of the song sounds exactly the same! So, although human beings may know nothing of the technical definition of "evenly tempered scales," they nevertheless use them naturally! In singing the same song at two different pitches, the ratios of the frequencies of corresponding tones of the two songs are exactly the same throughout the song. Or, the ratios of the frequencies at two different points in the song are the same, regardless of what pitch is used (e.g., bass, tenor, soprano, alto). A song is hence defined simply in terms of a series of ratios of the frequency of each tone of the song to the frequency of the starting tone.

The use of evenly tempered scales in music greatly simplifies the writing of music, since it does not matter what pitch (e.g., what octave) a musical instrument uses. It also dramatically simplifies the construction of musical instruments.

3. Guitar Fret Positions for an Evenly Tempered Scale

It was mentioned earlier that a flute with equally spaced holes produces tones on an evenly tempered scale. Now, let's discuss the guitar. From physics, it may be observed that the fundamental vibrating frequency of a taut string under constant tension is inversely proportional to the length of the string. Let p_i denote the position of the i-th fret of a guitar fretboard (i.e., the distance of the fret from the bridge), and let g_i denote the frequency of the string, when the string is pressed to the i-th fret and plucked. Then we have

$$g_i = c/p_i$$

where c is a constant, and

$$p_i/p_{i+1} = (c/g_i)(c/g_{i+1}) = g_{i+1}/g_i = 2^{1/12} = 1.059463094$$

since g_i and g_{i+1} are successive frequencies in the evenly tempered scale. Hence we have shown that the ratio of successive fret positions is $2^{1/12} = 1.059463094$, for an evenly tempered scale of twelve tones.

4. Why Use an Octave of Twelve Tones (Scale of Twelve Notes): Consideration of Two-Note Chords

So, if an octave of eight tones works well, in the sense that it can support a large variety of melodies, why does modern Western music use a musical scale in which the octave (range from a frequency to its double) is divided into a total of twelve tones: A A# B C C# D D# E F F# G G#? There are two reasons: one has to do with melody, and the other with harmony. (Melody refers to a (time-separated) sequence of tones. Harmony refers to a set of simultaneous tones.) First, by dividing the octave somewhat finer, the variety of melodies is substantially increased. With the simple flute, there was an obvious physical reason why the octave was divided into at most nine tones (viz., there are eight fingers to cover the tone-holes). With other musical instruments, however, this restriction does not hold. On a stringed instrument, such as a lute, a lyre, or a harp, it is possible to divide the octave into as many parts as desired. Obviously, there is a limit. Having 24 or 36 strings per octave would permit an almost continuous variation in tones over the range. But increasing the number of notes per octave would introduce serious problems, with the benefit of a hardly noticeable increase in the resolution of the scale. Music would be more difficult to learn and to play (and to record and read). Musical instruments would be more expensive to produce and maintain (replace strings, keep in tune).

So, the question remains, how many tones should there be to an octave? The more tones, the greater the melodic and harmonic complexity of the music. Consider an analogy with painting. A painting that consisted of a single color (e.g., black on white, or red on white) is not as “interesting” as one that involves many colors. A melody based on a single tone (e.g., a drum) is not very interesting. A painting based on just three primary colors (red, yellow, blue) is usually not very interesting or inspiring. Similarly, a melody based on only three notes would be rather dull. A painting based on an infinite variety of colors – as occur in nature – can be very interesting. Songs based on a twelve-note octave are definitely more interesting than songs based on an eight-note octave. The distinction is between “simple” melodies of a traditional flute, and more complex melodies of a modern twelve-note flute (or other modern instrument). At some point, however, there is little advantage in increasing the number of tones in the octave. Having twelve rather than eight yields a significant increase in the melodic character of music, but if there were sixteen or twenty-four tones instead of twelve, the variety of melodies would not seem that much more interesting, despite the introduction of a substantial increase in the complexity of the music and the musical instruments.

The complexity of the melodies that can be produced by musical instruments is just one dimension of music. The other major dimension is harmony. Harmony adds a tremendous new dimension to music. It is like the difference between stereophonic and monaural sound, or between quadraphonic and stereo sound – a really noticeable difference. Harmony can be produced by playing two different instruments at the same time, such as two flutes. Or, on a stringed instrument, such as a guitar, it can be achieved by playing two strings (of different notes) simultaneously. It is the issue of harmony that provides a definitive answer to the question of how many notes should an octave be divided into. The answer is definitely twelve – no more and no less.

Why twelve? Well, the reason is somewhat complicated. To understand the reason, it is necessary to understand a little about the physics of sound. We will focus our attention on sounds produced by strings, since it is the guitar (a stringed instrument) that is of interest here. (The theory is similar for sounds produced by open and closed pipes (flutes, pipe organs, brass instruments) or by bars (xylophone, marimba) or drums.) If a taut string is plucked, its vibration generates waves in the air, and it is these physical waves (periodic pressure variations) that our ear interprets as the physiological sense of sound. If the tension on the string is kept the same but the length of the string is halved, the frequency of the sound will double. Although an instrument such as a tuning fork produces mainly a single frequency (tone), when a string is plucked it produces – in addition to a main, or predominant, frequency – sounds over a continuous spectrum of frequencies. If the frequency spectrum (power spectrum, spectral density function) is plotted, you will see most of the energy concentrated near a single frequency, called the "fundamental" frequency, or first harmonic, of the string. You will also see, however, substantial amounts of energy near frequencies that are integral multiples of the fundamental frequency – double, triple, and so on – and relatively little energy at frequencies between the integral multiples. These other main frequencies are called the higher harmonics of the fundamental frequency – the second harmonic, the third harmonic, and so on. Most of the sound energy is in the fundamental frequency, with less and less energy in the higher and higher harmonics (and relatively little energy between the harmonics). Relatively little energy is in harmonics past the third. The important thing to remember about harmonics is that they are integral multiples of the fundamental frequency.

Although there is relatively little energy distributed between the harmonic frequencies, it is very important. It is how the energy is distributed over the entire frequency spectrum (i.e., the shape of the spectral density function) that determines the distinctive quality of the tones of various musical instruments and the voices of human beings.

The energy present in the various harmonics can be affected by plucking the string at different places. If the string is plucked in the middle, most of the energy is in the fundamental frequency. If you place your finger lightly touching the exact middle of the string and pluck it one-quarter of the distance from the end, most of the energy will be in the second harmonic. The sound that is generated will be an octave higher than the fundamental tone. It is as if two strings half the length of the original string were plucked. If you look carefully, you may be able to see the two halves vibrating separately, with no movement at the middle of the string. Similarly, if you place your finger lightly at a point one-third away from the end of the string and pluck it gently, you will generate mainly the third harmonic (i.e., the string is vibrating in three separate parts). Whenever the string is plucked in usual fashion, as in playing the instrument, lots of higher harmonics will be generated, with most of the energy in the fundamental and lower harmonics. The physical manifestation of harmonics on a string may be seen by means of the familiar high-school physics experiment in which a taut string lying on a sheet of paper covered lightly with sand is plucked. The wavy lines of the various harmonics will show up in as a pattern in the sand.

Harmony is produced by sounding two or more different tones at the same time. As everyone knows, however, some combinations of tones are very pleasing to the ear, others are less pleasant but interesting, and some are downright unpleasant. In general, two tones that are very close together in frequency sound unpleasant, or “discordant.” Tones that are far apart are generally pleasant, or neutral, or sound mildly strange. They are referred to as “concordant,” or “imperfectly concordant.” To understand what combinations of tones are concordant, it is necessary to understand harmonics.

Combinations of tones whose frequencies are integral multiples of each other (i.e., octaves apart) sound pleasing to the ear. Even tones whose frequencies are simple fractions of each other (e.g., the frequency of one is 1.5 times the frequency of the other) sound pleasant together. Other combinations of tones may sound all right together, or they may sound terrible. Whether two tones sound pleasant or unpleasant (concordant or discordant) depends on the mathematical (or physical) relationship between their frequencies, in particular their “harmonic relationship.” If the harmonics of one tone are harmonics of the other, the sounds will tend to reinforce each other, and they will sound pleasant together (i.e., “harmonious”). Note that since little energy is in the higher harmonics, it is mainly the relationship of the first two or three harmonics (of different tones) that determines how combined tones sound together. If any of the (first few) harmonics (including the fundamental) of different tones are close to each other (but do not match perfectly), the combined sound will “interfere.” “Beats” (regular fluctuations in volume) will be heard, and the sounds will not sound pleasant together. If the harmonics of one are not harmonics of the other, and the harmonics are not near in frequency, then the tones will sound all right together. They may not sound particularly harmonious, but they will not sound discordant, either. To summarize, tones and their harmonics must match almost exactly, or be quite different in frequency, for them to sound concordant.

Two tones that are one or more octaves apart (i.e., one is double the frequency of the other) sound pleasant together. Two tones of which the frequency of one is 1.5 times the frequency of the other will also sound pleasant together. For example, the tones A220 and E329.63 sound good together because the second harmonic of E329.63, which is E659.26, is (approximately) the third harmonic of A220, i.e., A660. The sound waves do not “interfere” with each other to produce a highly irregular mix of frequencies or beats. Quite the contrary, since the harmonics match, the tones “reinforce” each other (i.e., the peaks of the frequency spectra of the two tones match up closely in several places, so that the combined sound has substantially more energy near these frequencies). The sound is concentrated in several low harmonics, and these frequencies are not so close to each other that they interfere. The result is a sound that is very harmonious and powerful. It is an even (level, constant) sound, without noticeable beats or disharmony.

The simultaneous playing of two single-frequency tones that are close in frequency produces a combined sound that has a variety of frequencies, including those at the frequencies of the two tones, their sum, and their difference. In actual music, there is no such thing as a single-frequency tone. The closest thing to this is a tone that has most of the energy concentrated near a particular frequency. Clear-sounding notes, such as from a flute, will have most of the energy at the fundamental frequency. Complex-sounding notes, such as from a stringed instrument or brass instrument will have much energy at the fundamental frequency, but also a substantial amount of energy distributed over other frequencies, mainly the harmonics. For a single-frequency ("pure") tone of frequency f, the expression for the amplitude y(t) of the tone at time t is

$y(t) = \sin(2\pi ft)$.

For two theoretical single-frequency ("pure") tones of frequencies f1 and f2 and the same amplitude, the expression for the amplitude y(t) of the combined tone at time t is

$y(t) = \sin(2\pi f1t) + \sin(2\pi f2t) = 2 \sin(2\pi(f1+f2)/2) \cos(2\pi(f1-f2)/2)$.

That is, the frequency of the combined tone is a function of the sum and the difference of the frequencies. For two tones that are close together in frequency (e.g., by a few hertz), the combined tone can be heard to be comprised of two frequencies. The first frequency is (f1+f2)/2, or the average of the two frequencies. The second frequency is (f1-f2)/2, or half the difference in frequency of the two tones. Hence, when tuning the guitar, if two strings that are almost the same pitch are plucked, we hear the average pitch of the two notes, modulated by a frequency that is half the difference in the two pitches. The modulation is referred to as "beats," and it can be heard very distinctly as a fluctuation in volume. The number of beats heard per second is half the difference in frequency of the two strings. As the frequency of the string being tuned approaches the frequency of the other string, the number of beats per second decreases. When the number of beats per second is zero, the two strings are in tune. At this point, (f1+f2)/2= f1, f1-f2=0, the cosine term becomes 1, and the sine term becomes $\sin(2\pi f1)$.

On the guitar, tones that are one step apart (on the twelve-tone scale) sound terrible when played simultaneously – they are very discordant, and you can hear the interference beats clearly. With tones that are two steps apart, you cannot hear beats clearly, but the combined sound is somewhat discordant. Tones that are three steps apart sound all right together – not discordant. Tones that are five or seven steps apart sound particularly harmonious, for reasons that will become clear later.

With respect to the issue of deciding what tones (and how many tones) to include in a scale (i.e., in an octave), the matter of harmony plays a central role. It is desirable to have a selection of tones on the scale that will allow harmonious combinations. Specifically, it is desirable to have (at least some) notes whose harmonics match. If that is done, then it will be possible to construct truly beautiful-sounding chords from the notes of the scale (tones with harmonics that are well-matched and reinforcing). Otherwise, it will be possible to form chords that are not discordant, but they will not sound unusually great.

The question that we are addressing at the moment is how many tones will be selected for an octave. The answer to the question of how many tones is best will be restricted to the class of all tone-sets of evenly tempered scales, i.e., scales for which the ratios of the frequencies of successive tones of the scale are equally spaced in the logarithms of the frequencies (that is, the "interval" (or distance) between every two next-adjacent tones is equal on the logarithmic scale). In the case of the simple flute, the holes are equal distances apart. In the case of the guitar, the ratio of string lengths defined by the frets for any two next-adjacent frets is the same (for all selections of next-adjacent frets). (As discussed earlier, that ratio is the twelfth root of two, or 1.059463094.) The reason for this restriction to evenly-tempered scales is so that the relationship among the tones will be exactly the same at every place in the scale. This means, among other things, that a melody will sound exactly the same (apart from being higher or lower in pitch) no matter where it is played on the scale. It also means that a particular chord (selection of tones specified distances apart) will sound the same (apart from being higher or lower pitch). To allow tones of varying distances would cause all sorts of problems. A practical guitar could not be built under this scheme, since the fret locations would to be different for different strings. In constructing a musical piece, it may well be that the notes used in the piece are a subset of non-equally-spaced tones (e.g., eight notes out of twelve tones for Western music), but it would be problematic for the instrument to be constructed using non-equally-spaced tone intervals.

Note that we are addressing solely the issue of how many "equally spaced" tones (on the logarithmic scale) will be spread over an octave, not the number of notes per octave on which melodies (songs) will typically be based (which will be addressed later). In Western music, twelve tones define an octave, but most melodies are based on selections (sets) of eight notes, which are referred to as "keys" (or scales of a given key name). The tones of the 12-tone octave are equally spaced (on a logarithmic scale), but the tones of the selected eight-tone subset (scale or key) on which a music piece is typically based are (obviously) not equally spaced.

The reason why the best number of tones (equally spaced in the logarithms) for an octave is twelve is, briefly, that it produces chords of better harmony than any other number (less than 29). This could be demonstrated experimentally by building harps with varying numbers of strings per octave, and playing a variety of chords on them. It could also be demonstrated with a series of trombones (since the tones of the scale are determined simply by moving the slide). This paper will show mathematically why this result holds.

To simplify the problem a little, let us restrict it to the case of "triad-based" chords. These are chords that consist of three different notes, each of which may occur any number of times in any octave. For example, the familiar C chord consists of the notes C E G, where each note of the chord may be repeated any number of times (from other octaves). Chords based on just two notes are not very interesting. Chords based on four or more different notes will be addressed after the problem is solved for triad-based chords.

Let us divide the scale into n tones, and name the notes as 0, 1, 2,…,n-1. (The notes in the next higher octave will have names n, n+1,…,2n-1.) Let us denote the frequency of note i as fi. Then (because equal tone intervals correspond to equal intervals on the logarithmic frequency scale) we have the relationship

$$fi = f0\ 2^{i/12} = f0 \exp((i/12)\ln 2) .$$

Now let us consider a chord consisting of any two or more tones within an octave. (The best sounding chord from the viewpoint of "reinforcement" will be the one containing two tones an octave apart. That is excluded by the restriction that the tones be within an octave.) From consideration of harmonics, the best sounding chord will be one in which the second harmonic of the second (higher) tone is the third harmonic of the first tone. The second harmonic of the second tone cannot be the second harmonic of the first tone, since the second harmonic is an octave above the fundamental – both notes would be the same.)

Let us denote the first note of the two-note chord as a and the second note as b. The frequency of the first note of the two-note chord is denoted as fa, and the frequency of the second note is denoted as fb. If the second harmonic of the second tone is the third harmonic of the first tone, then the following relationship holds:

$2fb = 3fa$ (or $fb = 1.5 fa$).

Since

$fi = f0 \exp((i/n)\ln 2)$,

we have

$fa = f0 \exp((a/n)\ln 2)$

$fb = f0 \exp((b/n)\ln 2)$,

so the equation $2fb = 3fa$ yields

$2 \exp((b/n)\ln 2) = 3 \exp((a/n)\ln 2)$

or

$\ln(3/2) = ((b - a)/n)\ln 2$

or

$(b - a)/n = \ln(3/2) / \ln 2$.

Note that the solution depends only on the note distance between the tones, and not on the position in the scale. (This follows since the notes are equally spaced on the logarithmic scale – the ratio of the frequencies of any two notes is the same as for any other two notes the same distance apart.) Hence we may without loss of generality simply designate the first note of the chord as the first note of the scale (i.e., set $a = 0$), and obtain

$b/n = \ln(3/2) / \ln 2 = .584962501$.

So, the question of determining n (the number of tones in the octave) reduces to determining for what value of n the ratio b/n is very close to .5850, where b is any number between 1 and n. For values of n from 3 to 24, the nearest values of this ratio to .5850 are:

1/3 = .33; 2/3 = .67; 2/4 = .5; 3/4 = .75; 2/5 = .4; 3/5 = .6; 3/6 = .5; 4/6 = .67; 4/7 = .57; 5/7 = .71; 4/8 = .5; 5/8 = .62; 5/9 = .56; 6/9 = .67; 5/10 = .5; 6/10 = .6; 6/11 = .55; 7/11 = .64; 7/12 = .5833; 8/12 = .67; 7/13 = .54; 8/13 = .62; 8/14 = .57; 9/14 = .64; 8/15 = .53; 9/15 = .6; 9/16 = .56; 10/16 = .62; 10/17 = .5882; 11/17 = .65; 10/18 = .56; 11/18 = .61; 11/19 = .5789; 12/19 = .63; 11/20 = .55; 12/20 = .6; 12/21 = .57; 13/21 = .62; 12/22 = .55; 13/22 = .5909; 13/23 = .57; 14/23 = .61; 14/24 = .5833; 15/24 = .62.

The closest value occurs for n=12 and b=7, yielding 7/12=.5833 (the same value occurs for n=24 and b=14, but there is no point in using more strings when less will accomplish the same result). The value .5833 is very close to the desired value .5850. It corresponds to a frequency of

f7 = f0 exp((7/12)ln2) = 1.498307077 f0.

That is, the seventh note of the twelve-tone scale is within one-tenth of one percent of the desired value of 1.5. The frequency of the second harmonic of the seventh note is

f19 = f0 exp((19/12)ln2) = 2.99661454 f0.

(The preceding may seem to be a rather complicated way of determining the value of n for which there is a tone of frequency very close to 1.5 times the root tone of the scale. To do this directly, however, involves more computation than above, since it is necessary to calculate the relative frequencies of the notes of each scale. The preceding method involves simply calculating simple fractions and comparing their values to .585.)

If we go beyond the value of n=24, the next value of n having a closer value of b/n to .585 than b=7, n=12 occurs for b=17, n=29, i.e., b/n=17/29=.5862. This value differs from .585 by only .0012, whereas 7/12 differs by .0017. This is a modest improvement, but it would not justify the complication of increasing the number of tones of the scale from 12 to 29 if 12 works well, as we shall shortly show that it does.

In summary, the best-sounding pair of two notes within the same octave is that for which the frequency of one note is 1.5 times the frequency of the other. For no scale up to 24 equally spaced tones does this happen exactly, but it almost happens for the scale of 12 equally spaced tones. The second harmonic of the seventh tone on a twelve-tone scale is almost exactly the third harmonic of the first tone of the scale. (That is, the frequency of any note seven tones up from another (on an equally tempered 12-tone scale) is almost exactly 1.5 times the frequency of the first note.) This means that any two notes that are seven tones apart on the twelve-tone scale will sound really good together. Not only really good, but just about the best that any two notes (within an octave of each other) sounded together can sound.

Let us look at an example, in which the first tone of the scale is A220. The twelve-tone scale is exactly the one used in Western music, and so the seventh tone of the scale is E329.6275569 (= 220 x 1.498307077). If f0 is A220, then its third harmonic is 660, and the second harmonic of the third note of the scale is 2 x 329.6275569 = 659.2551138 hertz. (This could also be derived from 2.99661454 x 220 = 659.2551988 hz.) This is less than one vibration per second from the third harmonic of A220 (660 hz)!

Since b=7, n=12 works so well, there is little point in considering the slightly closer solution b=17, n=29 (which would require an octave of 29 tones rather than 12!).

All that matters is the interval between the two tones, not the absolute frequency. Hence, for a given note (e.g., A), the seventh note above it (E) or below it (D in the octave below) will sound good together.

To see just how good this chord is compared to other possibilities (i.e., chords from evenly tempered scales based on other than 12 tones), let us examine how close the second harmonic of tones of other scales can be to the third harmonic of the first tone. These are given by the formula

$fx = f0 \exp(x \ln 2)$,

where x is the closest fraction to .585 given in the list of fractions (ratios) given above. We have, for the two next-closest ratios:

$2f8/14 = 220 \exp(8/14 \ln 2) = 653.84$

$2f10/17 = 220 \exp(10/17 \ln 2) = 661.5$.

The first one is off by six hz, and the second by 1.5 hz. In the first case – the fourteen-tone octave, a difference of six hz would be quite enough to make a noticeable difference. The quality of the two-tone chord consisting of the fundamental and seventh-higher note of a 14-tone octave would not sound that great. In the second case – the seventeen-tone octave – the second harmonic of the tenth note above the fundamental is quite close to the third harmonic of the fundamental. The difference is 1.5 hz. But this is twice the difference for the result for the 12-tone scale (about .74 hz difference). And for a 17-tone octave. There is little point in considering dividing the octave into more tones, if the best two-tone chord is not as good as the best one for the twelve-tone octave.

5. Consideration of Three-Note Chords

There is one reason, however, for considering it further. It may be that the addition of a third note to the two-note chord works better for the 17-tone octave than for the 12-tone octave. It turns out, however, that the (physiological) quality of the sound of three-tone chords is affected more by the spacing of the third tone from the other two, than on whether its harmonics match the harmonics of the other two notes. Furthermore, it is possible to find third notes for the chord that sound quite good on the twelve-tone octave. For this reason, there is no practical reason for considering the 17-tone octave any further.

Let us now consider, then, the issue of adding a third tone to the “best-sounding” two-tone chord. In view of the above results, we shall restrict consideration to the case of the 12-tone octave. At this point, it is helpful to present the discussion in terms of a specific twelve-tone scale, namely the one starting at A220 and going to A440. (The theory could be expressed in terms of arbitrary frequencies, but it is a little more intuitive to use familiar frequencies.) Note that the frequency A440 is an international standard.

The frequencies for all other notes on the 12-tone scale are fractions (except, of course, for As at other frequencies, such as A220, A880).

Using the formula

fi = 220 exp((i/12) ln2),

the frequencies of the other notes may be calculated (for i =1 to 12):

A 220 hz
A# 233.0818808
B 246.9416506
C 261.6255653
C# 277.182631
D 293.6647679
D# 311.1269837
E 329.6275569
F 349.2282314
F# 369.9944227
G 391.995436
G# 415.3046976
A 440.

To put things in perspective, middle C on the piano is 261.63. A440 is the second space from the bottom on the treble clef. Middle C (261.626) is the first ledger line below the staff (stave) on the treble clef, and the first ledger line above the staff on the bass clef. Middle C is the first fret, second (narrowest) string on the guitar. Note, however, that guitar music is written an octave higher than it is played (so that most of the notes occur on the staff rather than on ledger lines below it).

Let us consider the chord for which the first note is A220. We have seen that the best-sounding two-tone chord is AE. Chords based on three tones sound better (richer, fuller) than chords based on just two, so there is no question about wanting to add the third tone. The issue to address is what the third tone should be. In order for the chord to sound good, the distance of the third tone should be as far as possible from A and E. There are four choices: C, C#, F# and G.

Now F# and G are just two tones away from E or A, so they are eliminated. Tones that are an interval of just two (tones) apart (on a twelve-tone scale) sound noticeably discordant. Hence there are just two choices: C and C#. The chord ACE is called A minor. The chord AC#E is A major. (We will address the issue of naming chords later.) If you play these two chords, you will find that the A major chord sounds much more harmonious than the A minor chord. The reason for this is that the frequency distances of C from A and E are 41.6 and 68, whereas the frequency distances of C# from A and E are 57.2 and 52.4. That is, C# is about halfway between A and E on the frequency scale, whereas C is much closer to A than to E.

It would appear from the tone distances that either chord would sound as good, since in both of them the middle tone is three tones away from one of the other tones of the chord, and four tones away from the other tone of the chord. Because tones are logarithms of frequencies, however, the frequency interval increases as the tones increase, and chords having a spacing of 4 tones / 3 tones between the chord tones sound more harmonious ("balanced," stable, more concordant) than those having a spacing of 3 tones / 4 tones.

It might be wondered why the best sounding three-tone chord is not comprised of a tone plus the tones seven tones above and below it (i.e., the two tones that are closely harmonically related to it). In this example, A, E, and D (octave below). The reason is that while A sounds good with E or D(octave below), D and E are only two tones apart. In other words, A and E sound good together, A and D(octave below) sound good together, but D and E in the same octave do not sound good together.

To summarize, there are two different tones that may be added to the two-tone chord to produce a three-tone chord. If the fourth tone from the fundamental is added, the chord is called a "minor" chord, and if the fifth tone from the fundamental is added, the chord is called a "major" chord.

If a fourth note is added to a three-note chord, it cannot be more than two tones away from any other note of the chord. As was discussed earlier, tones that are a distance of two tones apart on a 12-tone scale are slightly discordant. The level of discordance drops, however, if the added tone is above all the others (rather than among them).

6. Other Considerations: Wind Instruments

At this point, we have shown that the twelve-tone scale produces very harmonious chords. This fact is probably reason enough to adopt a twelve-tone scale. It is more difficult to construct instruments that produce twelve tones per octave, however, than those that produce a smaller number, such as eight. Because of this, it is of interest to address whether there are any other factors that affect the choice of the number of tones per octave. It turns out that there are.

Consider the case of brass wind instruments, such as the bugle, trumpet, cornet, flugelhorn, French horn, trombone (tenor or bass), alto horn, baritone horn, sousaphone and tuba. How many tones per octave are needed to represent the tones that they produce? To answer this question requires an examination of the kinds of tones that these instruments produce. We shall look at the tones that are produced in "open" position, i.e., without using the valves or slide.

The brass instruments listed above are Bflat instruments. That means that when played in the open position (without pressing any keys or moving the slide), the resonant frequency at the lowest playable tone (i.e., the fundamental frequency) without pressing any valves or sliding the slide is Bflat. Tones of higher frequency are produced by vibrating the instrument at other "resonant" frequencies, which are harmonics of the fundamental frequency (corresponding to a given configuration of valves or slide position). The tones that are produced, for example, on a Bflat (tenor) trombone are:

Fundamental, or first harmonic: very low ("pedal") Bflat (58.270 hz)
Second harmonic: low Bflat (116.541 hz)
Third harmonic: 174.811 hz. Close to F = 174.614)
Fourth harmonic: middle Bflat (233.082 hz)

Fifth harmonic: 291.352 hz. Close to D = 293.665)
Sixth harmonic: 349.623 hz. Close to F = 349.228)
Seventh harmonic: 407.893 hz. Closest (but not very close) to G# = 415.305)
Eighth harmonic: high Bflat (466.164 hz).

Middle Bflat is the first space above the staff on the bass clef. Low Bflat is the second line from the bottom. Pedal Bflat is the third space (on ledger lines) below the bass clef.

As a boy, I wondered why my music teacher told me that playing the D in first position on the trombone was not quite in tune, and that it was better played in fourth position. The reason is obvious from the frequency list given above. While the third harmonic of the fundamental frequency (F, in first position) is quite close to the correct frequency for F, the fifth harmonic (D, in first position) is over two cycles per second off the correct frequency for D. In this frequency range, this is enough to make a difference (in some cases).

The brass instruments are simply long tubes (bent into compact shapes) that have an oscillator at one end and a flared opening at the other end. Given this geometry, the lowest frequency at which the column of air in the tube resonates is a sound wave whose wavelength is half the length of the tube. For this lowest-frequency resonant wave, the relative pressure at the mouthpiece is a maximum when the relative pressure at the bell (the flared end) is a minimum. For the second harmonic, there is a full wavelength along the length of the tube (i.e., the relative pressure is a maximum both at the mouth and at the bell). For the third harmonic, there are one and a half wavelengths along the tube. And so on.

It is easy to determine which Bflat is the pedal tone. The length of a (tenor) trombone is approximately ten feet. The speed of sound in dry, sea-level air is 1052.03 + 1.106T, where T is the temperature in Fahrenheit. The "standard" value is 1188 feet per second. The formula relating frequency, wavelength, and speed of sound is:

$$s = f\,w$$

where s denotes speed, f denotes frequency, and w denotes wavelength. For the pedal tone, we have w = 2 (10) = 20 feet (since the tubing length is a half wavelength), so

$$f = s/w = 1188/20 = 59.4 \text{ hz},$$

which is close to the standard value of 58.270 hz.

If we wish to use the trombone or other brass instrument as a basis for making music, it is desirable (necessary) to have an equal-interval scale that can represent the various tones that these instruments can produce. And this is where the twelve-tone octave becomes necessary.

Suppose, for example, that we used an eight-tone octave (of equal tone intervals, not a major scale). The frequencies of the tones on this scale are given by the formula

$$f_i = f_0 \exp((i/8) \ln 2).$$

If f_0 is 58.2704702, then the frequencies of the tones are:

f0 = 58.270
f1 = 63.544
f2 = 69.296
f3 = 75.567
f4 = 82.407
f5 = 89.865
f6 = 97.999
f7 = 106.869
f8 = 116.541
f9 = 127.089
f10 = 138.591
f11 = 151.135
f12 = 164.814
f13 = 179.731
f14 = 195.998
f15 = 213.737
f16 = 233.082
f17 = 254.178
f18 = 277.183
f19 = 302.270
f20 = 329.628
f21 = 359.461
f22 = 391.995
f23 = 427.474
f24 = 466.164.

The problem that arises is that the tones of this eight-tone scale do not match the harmonics produced by the brass instrument. Unlike the twelve-tone scale, there is no tone close to F(174.614) or D(293.665). Hence the eight-tone scale cannot be used to represent the notes produced, for example, by a bugle or a "Roman" (no valve) trumpet. From the list of frequencies of the twelve-tone scale that was presented earlier, we see that the values of the all harmonics (produced by the brass instrument) except the seventh correspond very closely to tones of the 12-tone evenly tempered scale.

Hence there are two very good reasons for using a 12-tone evenly tempered (chromatic) scale. It enables chords that sound better than any other scale, and it can be used to represent music produced by brass (pipe) wind instruments (i.e., the tones of the twelve-tone scale are very close to many of the harmonics of the fundamental tone, and this is not true of scales based on smaller numbers of tones). Even scales with up to 24 tones per octave do no better.

So the issue of how many tones to divide an octave into is hence resolved. The answer is unequivocally twelve. It would appear that the Biblical entreaty to "sing unto the Lord with an instrument of twelve strings" had a basis in music theory!

B. The Issue of the Selection of Tones as a Basis for Musical Pieces (How Many, and Which Ones)

1. Some Basic Theory

We now turn to the issue of determining how many notes to use as a basis for musical pieces, and what that selection should be. In Western music that is usually eight, and the eight that are selected form one of two different patterns, called major and minor scales. These two patterns are based on the chords that were developed earlier.

Recall that the best two-tone chord used tones that were seven tones apart on the twelve-tone scale, and that there were two reasonable possibilities for the third tone – either the third tone of the scale (in which case the chord is called a minor chord) or the fourth tone of the scale (in which case the chord is called a major chord). Using the fourth tone produced a more harmonious sound. If it were not for chords (harmony), it would not matter a great deal how many notes were selected as a basis for music. As was discussed earlier, using a selection of eight provides a full, rich variety of (melodic) music.

The main reason for basing musical pieces on fewer than twelve tones is simplicity – if eight, for example, works well, why use all twelve? If use is to be made of the chords that were derived above, however, there is a reason why a particular selection of eight tones works very well. This brings us to the topic of chord relationships and progressions.

Before proceeding, it should be noted (perhaps it was obvious) that a chord that is based on three tones, such as CEG, can include these same tones from other octaves, and still sound good. This is done in music all the time. For example, the E chord on a guitar consists of the tones EBEA#BE, including tones from every one of the six strings and covering two octaves. Because of this fact, it is better to speak of the "notes" that define a chord, since a given note may occur several times (in different octaves). In other words, there may be more than three tones in a (triad) chord, but there are just three notes (in a triad chord).

For music to sound good, the harmony (chord) should match the melody in some way. Usually, music sounds good if, while a particular chord is being played, the melody uses some or all of the notes from the chord. In other words, the melody is to some extent based on the chord. Music would be rather boring, however, if we played only one chord for the entire piece. Just as melodies vary over the full range of the scale to be interesting, chords should, too. The question is, what chord should be played next.

To some extent, the melody will be a guide. The chord must match the melody, and as the melody changes, some chords will match (sound) better than others. On the other hand, it is possible to play a piece that consists entirely of chords (no melody). In this case, there is no melody to guide the transition ("progression") of one chord to another. Nevertheless, it sounds better to move from a particular chord to certain others. This is true because chords are related by the notes of which they are comprised. Recall that the basic two-tone chord was developed by including a tone and the seventh tone up from that tone. Because of the cyclical nature of the musical scale, it could just as easily have been the seventh tone down from that tone. The third tone added to the chord was intermediate between these two tones. The first tone of the chord is called the “root” of the chord. The chord is named after the root tone, e.g., the chord CEG is called the C chord.

For a given chord, there are two other chords that sound particularly good with it. Those are the chords whose roots are seven tones above and below the root of the given chord. Most pieces of music begin and end on the same note (the “tonic” note, or key note), and on the same chord (the tonic chord). In fact, quite short segments, or “phrases” of pieces, often start and end on the tonic note. In Western music, the key note, or tonic note, is the anchor for the whole piece. The music – both the melody and that harmony – keeps returning to the tonic note, and the piece ends on the tonic note, and generally on the tonic chord.

For example, consider the chord CEG. The two other tones that are closely related to C are G and F. G is the seventh tone from C (so the second harmonic of G is C) and C is the seventh tone from F (so the second harmonic of F is C). The three chords CEG, GBD, and FAC are closely related, and tied to the C tone, and it sounds good to move from one to the other. A piece sounds “complete” when it ends with the chord (and the note) on which it started, and so the music will move, or “progress” from C chord to F or G some number of times, returning to C from time to time, and eventually ending on C.

Songs do not have to use all three chords. If only two chords are used, it is usually the tonic chord and the chord whose root is seven tones above it (a "fifth" above, in the terminology of modern music, since it is five (alphabetic-name) notes above, counting both the starting and ending notes).

There is much more to the subject of chord progressions than will be addressed here. One additional item that will be mentioned is the topic of “seventh” chords. A “seventh” chord is a four-tone chord, in which the tenth tone of the scale is added to the other three (first, fifth, and eighth tones of the scale). (The reason it is called a “seventh” instead of a “tenth” derives from the method of naming notes, and will be addressed later.) A seventh chord sounds pretty good, but it sounds a little “unstable,” or “incomplete,” or discordant. There is a desire to remove the “seventh” (tenth tone) to make it more stable. Hence in songs, a seventh chord is often introduced just before moving to the final chord (root chord) of the song. For example, a progression might be C, F, G, G7, C (each played some number of times).

Of the two chords that are related to a “tonic” (or “key” or “root”) chord, the one that is seven tones above the tonic chord is called the “dominant” chord, and the other one (seven tones below) is called the “subdominant” chord. For example, if C is the tonic chord, then G is the dominant chord and F is the subdominant chord. (In modern music terminology, a chord whose root is seven tones above another is said to be a "fifth" higher, because it is five notes above, counting the root note as 1). For any key, the last note of the scale is called the “leading note.” It is so called because it often “leads into” the tonic note. The dominant chord contains the leading note, and hence it often leads into the tonic chord. If a “seventh” is added to a dominant chord, it creates a somewhat “incomplete” sound, that is not as solid, or complete (or harmonic) as the basic triad chord to which it is added (since the tones and harmonics of the chord notes are not as concordant as when it is omitted). In a sense, this chord “begs for” resolution – the music sounds “up in the air,” or unfinished. A perfect way to end the musical phrase is on a tonic chord preceded by a dominant seventh. The “incompleteness” caused by the addition of a seventh is resolved, and the presence of the leading note causes the chord to lead naturally into the tonic chord.

In addition to seventh chords, there are augmented and diminished chords, which serve much the same purpose as the seventh chords (variety, sense of incompleteness (imperfect concordance, or slight discordance), i.e., as leads to other chords). To summarize, there are basically three reasons for moving (or progressing) from one chord to another: (1) To introduce variety into the musical piece; (2) To “resolve” the chord by removing the slight discordance that may have been introduced by the inclusion of other notes, such as sevenths, ninths, and the like, or intervals of incomplete concordance; and (3) To return to the key chord, which is often done from a seventh chord containing the leading note to the key note.

An interesting thing to observe is that the three chords CEG, GBD and FAC include a selection of seven tones of the scale: CDEFGAB. These seven tones span the full octave (including the high C) rather evenly, and while they do not include every tone of the 12-tone chromatic scale, songs constructed from them sound, in a sense, complete, and they include a rich range of melodies. The sequence of tones in this scale is C (fundamental), up two tones to D, up two tones to E, up one tone to F, up two tones to G, up two tones to A, up two tones to B, and up one tone to C. This pattern of tones – 2,2,1,2,2,2,1 – is called a major scale pattern.

If we base the musical piece on the minor chord (fundamental plus fourth tone up plus seventh tone up) then there are still three related chords, but the notes that they represent from the chromatic scale form the pattern 2,1,2,2,x,x,x, which is called a minor scale pattern. (There are several minor scales, hence the x’s – we will return to this later.)

In some music books, and in some countries, the two preceding scale patterns are referred to using the terms “tone” and “semitone,” denoted by T and S. For example, the major scale pattern is denoted as TTSTTTS, and the (ascending melodic) minor scale pattern is denoted as TSTTTTS. That practice represents somewhat of a corruption of the term “tone,” which linguistically refers to a sound (frequency level), not a sound *interval*. In that use of the word “tone,” a “tone” refers to an interval of two tones on the twelve-tone scale, and a “semitone” refers to an interval of one tone on that scale. Other books use the term “step” and “half-step” (so that the major scale pattern is SSHSSSH). That is somewhat better than corrupting the word “tone,” but not much. It implies that the octave is divided into six full “steps,” when in fact it is divided into twelve tones. Since there is nothing magical about a “step” of two tones, and since the scales on which music is based involve eight tones, not six, this is nothing more than confusing. If the terminology “step” (S) and “two steps” (T) were used, less confusion would arise (major scale pattern TTSTTTS). It is much more descriptive to use numbers to represent the sequence of tones, as in 2,2,1,2,2,2,1 (where 1 represents a step of one tone up the scale, and 2 represents a step of 2 tones up the scale).

2. Some Examples

The example above was in the key of C, in which the notes of the major scale involve none of the tones having sharps or flats in their names. The following is an example in which the root note is A. In this case, and in all other cases, the major and minor scales involve one or more notes with sharps or flats in their names.

Before presenting the example, it is noted that the musical notation used today is quite complicated. It is complicated because only seven letters are used to name the twelve tones. Seven tones hence have simple names (ABCDEFG) and with suffixes (sharps and flats) used to denote the other five. Although the notation is complicated, it does help the player of an instrument in a way, since to keep track of which eight notes are used as the basis for a piece he must keep track simply of which tones are "suffixed," rather than try to remember, without any assist, which of the twelve tones are selected for the piece's scale. (It also results in a staff of fewer lines.) The complications evidently arose because letter names (ABCDEFG) were used for the seven tones of one of the major scales (C major), and it was evidently desired to use variations on these same names for all of the twelve tones of the full chromatic scale. The unmodified names were used for the scale of C major: CDEFGAB. Since the major scale pattern is 2212221, this means that the other five tones of the full chromatic scale must fall in the places marked with X: CXDXEFXGXAXBC. The name modifications were effected by using sharps or flats: a sharp raises a tone to the next higher tone, and a flat lowers it. This means that the names could be CC#DD#EFGG#AA#BC or CDfDEfEFGfGAfABfBC, where suffix # denotes sharp and suffix f denotes flat. Depending on which note is the fundamental, or root, of the scale, the eight notes of all major scales other than C will involve some sharps or flats. It is required that every letter be used exactly once in the scale, which is equivalent to the requirement that a scale involve only sharps or flats (or none), but not both.

Now for the example. Let us list the notes of the (full twelve-tone) scale twice: once using sharps and once using flats:

A A# B C C# D D# E F F# G G#

A Bf B C Df D Ef E F Gf G Af .

Now suppose, for example that we wish to find the major chords and major scale based on the first note, A. As we have done previously, let us denote the first note of the scale as 0, and the others 1, 2, …, 11:

0 1 2 3 4 5 6 7 8 9 10 11 .

The pattern of tones for a major chord is fundamental + 4 tones up + 7 tones up, or 0 4 7. The patterns for the other two closely related chords are 7 11 14=2 and 5 9 12=0. (We are dealing with a cyclic scale, and it is customary when the octave does not matter to refer to notes modulo 12, i.e., if a note has a name greater than 12, subtract 12 from that name and use the residue as the name.) The pattern of tones for a major scale (from the step pattern 2 2 1 2 2 2 1) is 0 2 4 5 7 9 11 (13=0). (Note that the chords 0 4 7, 7 11 2, and 5 9 0 include all the notes 0 2 4 5 7 9 11 of the scale.) Hence, in terms of "modern" music notation, the chords of the key of A major are:

Chords: A C# E, E G# B and D F# A

Scale: A B C# D E F# G# .

Hence the scale of A major has three sharps, F#, C#, and G#.

One more example. Consider the key of Bf (Bflat) major. Following the same patterns as above, we have:

Chords: Bf D F, F A C and Ef G Bf

Scale: Bf C D Ef F G A .

Hence the scale of Bflat major has two flats, Bf and Ef.

3. Conventions for Naming Note Intervals and Chords

In modern music, there are a number of different types of chords – more than just major and minor chords. There are also seventh chords, augmented chords, and diminished chords (and others). These additional chords are useful in expanding the variety of harmonic sounds, and providing additional “sequences” in chord progression. (If all songs involved only the three basic chords discussed above, music would not be nearly as interesting as it is.) The names of the chords are based on the terminology for naming intervals between tones. It is helpful to understand this terminology.

Because only seven letters are used in modern music notation as the basis for naming all twelve notes of the scale, and because only five of the notes have suffix names (sharps or flats), the intervals between unsuffixed notes vary. For example, there is no tone between B and C, or between E and F, whereas there is a tone between all of the other unsuffixed note names (e.g., between A and B is A# or Bf). Because of this, the terminology (in modern music notation) for referring to the notes in chords is a little complicated, since it refers to distances between note names, not between tones of the twelve-tone chromatic scale.

It is also a convention that the first note of a scale is referred to as 1, and that the note intervals are measured by counting both the starting note and the ending note. This is an incredible departure from measurement procedures in all other fields, where an interval is measured as the difference between two points a scale. For example, the distance interval between the one-inch mark and the five-inch mark on a ruler is four inches. In music, the interval between the A note and the E note is called a “fifth,” not a “fourth.”

For example, the major chord pattern 0 4 7 is referred to as root, major third, fifth. As a specific illustration, in the (major) chord CEG, the G is referred to as a “fifth” above C, because it is the fifth letter above C, counting C as the first. A (perfect) “fourth” is a note five tones above another. Note that “fifths” and “fourths” are musically identical (in terms of distance from the root), since if a note is a “fifth” (seven tones) above another note, it is also a “fourth” (five tones) below it. Fourths and fifths are referred to as “perfect” because they occupy the same position in both the major and minor scale patterns.

A problem arises with “seconds” and “thirds,” however, since their relative positions in the major and minor scales are different. A major third represents an interval between tones that are four tones apart. A minor third represents an interval between tones that are three tones apart. A major second represents an interval between tones that are two tones apart, and a minor second represents an interval between tones that are one tone apart. There are similar terms for sixths and sevenths. There are other terms, such as augmented and diminished fifths and fourths.

This is a little complicated, so I will summarize. The reason for spending time on the subject of naming chords and their relationships to each other is that to be able to change keys easily while playing by ear it is important to understand which chords are used in a piece, and what their relationship to each other is. The terminology of modern music is complicated, but we are stuck with it.

I will state the technical definitions of the interval names, and then provide some additional examples. Although this article will be restricted to major and minor chords, I will, for completeness, include definitions of the all note intervals, not just the ones that refer to these two chord types.

Minor second: Two notes one tone apart (i.e., they are adjacent tones on the 12-tone chromatic scale). Example: ABf
Major second: Two notes two tones apart. Example: AB
Minor third: Two notes three tones apart. Example: AC
Major third: Two notes four tones apart. Example: AC#
Diminished fourth: Two notes four tones apart. Example: ADf
Perfect fourth: Two notes five tones apart. Example: AD
Augmented fourth: Two notes six tones apart. Example: AD#
Diminished fifth: Two notes six tones apart. Example: AEf (also called a “tritone”)
Perfect fifth: Two notes seven tones apart. Example: AE
Augmented fifth: Two notes eight tones apart. Example AE#
Minor sixth: Two notes eight tones apart. Example AF
Major sixth: Two notes nine tones apart. Example AF#
Minor seventh: Two notes ten tones apart. Example AG
Major seventh: Two notes eleven tones apart. Example AG#
Diminished octave: Two notes eleven tones apart. Example AAf
Perfect octave: Two notes twelve tones apart. Example: AA
Augmented unison: Two notes one tone apart. Example AA#

The above list is the more common intervals. I have not included every possibility in the above list. For example, there are also diminished and augmented seconds, thirds, sixths and sevenths. These are enharmonic equivalents of major and minor intervals already in the list. Note that there are alternative names for the intervals. Which name to use depends on the key of the piece in question. There is a convention that a scale (key) is defined using only flats or sharps, not both, and every letter (ABCDEFG) is used exactly once. Hence, in the key of F (one flat, Bf), we do not refer to C#, or else there would be two “A” notes, A and A#, in the scale (AA#CDEFG), and no “B” note (as in ABfCDEFG).

The terms “concordant” and “discordant” were introduced earlier. Intervals of 4, 5, or 8 are called “perfect concords.” Major and minor thirds and sixths are called “imperfect concords." All other intervals (seconds, sevenths, augmented, diminished) are called "discords."

The terms "major," "minor," "augmented," and "diminished" that refer to tone intervals are used for naming chords and keys. Hence, the terms "minor" and "major" used to describe note intervals correspond to intervals on the minor and major scales. For example, a minor third is the first and third notes on a minor scale, and a major third is the first and third notes on a major scale. (Recall that a minor scale is a selection of any eight notes using a minor scale pattern 2122xxx, and a major scale is a selection of any eight notes using the major scale pattern 2212221.)

A major chord is one that includes the root note, a major third (interval), and a perfect fifth (interval). A minor chord is one that includes the root note, a minor third, and a perfect fifth. A seventh chord is one that includes a seventh. A diminished chord includes a diminished interval, and an augmented chord includes an augmented interval.

A fifth, or perfect fifth, is a difference of five notes on a major scale, counting the first and last notes, and starting the counting at the beginning of the scale pattern. It is hence any two notes that are an interval of seven tones apart (NOT counting both the first and last tones, but the number of tones it takes to move from one to the other). Note that a musical fifth has nothing to do with the mathematical fraction fifth. A musical "fifth" is not a fifth part of anything. It simply means the fifth note up or down from another, counting the first note as "one" and moving in steps defined by the major scale pattern (two steps plus two steps plus one step plus two steps; for example the fifth of A is E: A itself is the first step; A to B (2 tone steps) is the second step; B to C# (2 steps) is the third step; and C# to D (1 step) is the fourth step; and D to E (2 steps) is the fourth step).

The terminology for naming chords involves the interval names listed above, but it is a little complicated. Points to note are the following:

1. A major chord may or may not be referred to as such, e.g., A or Amaj.
2. A minor chord is always referred to as such, e.g., Amin.
3. A seventh chord is always a minor seventh chord, e.g., A7 is a minor seventh (interval) added to A; Amin7 is a minor seventh added to Amin. In other words, the term "minor" in Amin7 refers to the A, not to the seventh.
4. A major seventh chord is a major seventh added to a major chord, e.g., Amaj7 is a major seventh added to A. In other words, the term "major" in Amaj7 refers to the seventh and not to the A.

4. The Circle of Fifths

Note that every set of three closely related chords (with roots seven tones apart) contains two chords of another set of closely related chords. If we jump through the twelve-tone scale in steps of seven tones, we will eventually hit every tone. Every sequence of three tones will represent a set of closely related chords. Starting with C, this chord sequence is:

C G D A E B(Cf) F#(Gf) Df(C#) Af Ef Bf F C .

In the terminology of modern music, these successive chords (whose roots are seven tones apart) are each a “fifth” apart. For example, C D E F G (including the starting and ending notes). Or, A B C D E. Or, F# G# A# B C#. Note that to find the fifth of a given note we use the major scale pattern (2212221) in moving through the note names A A# B C C# D D# E E# F G G#.

(A note on notation: In describing chords, we will typically run the note names together, e.g., CEG for the C chord. In describing sequences of chords, as in the case of a chord progression in a song, we will typically include hyphens between the chord names, e.g., the chord progression C-F-G-C (or, as will be discussed later, I-IV-V-I). Spaces or commas may be used as separators in either case (e.g., the chromatic scale would be listed as A A# B C C# D D# E E# F F# G G#, and an unrelated list of chords would typically be listed with spaces, as in C D E F G. Hyphens will typically be used to separate numbers, e.g., as in note sequences 0-4-7 or 4-8-11, unless the numbers are all single-digit, e.g., as in the major scale pattern 2212221. A scale may be concatenated without separators, e.g., CDEFGABC. In any event, it should be clear from context whether we are talking about notes or chords (or tones or tone intervals).)

The chord sequence above is usually displayed as a circle, and is called the “circle of fifths” (or, sometimes, the "cycle" of fifths). It is useful to memorize it, because then you automatically know what chords are included in each key. For example, the key of G includes the chords C, G, and D. In fact, it is not necessary to memorize the circle of fifths at all, since the three related chords can be determined in an instant. Simply list the chords alphabetically from the root chord for which you want the other two related chords, and the desired chords are the fourth and fifth higher letters. For example, C D E F G, so F and G are the chords harmonically related to C (i.e., they are seven tones apart, or “fifths” apart, in terms of the modern music notation). The only problem is determining whether the chord is sharped or flatted. For chords from F to B (F C G D A E B), that is no problem – they are all naturals. From that point on, they are all sharped (or may be represented as the flatted next-higher note). So the complete circle is F C G D A E B F# C# G# D# A# F, or F C G D A E B Gf Df Af Ef Bf F. When shown as a circle, C is usually placed at the top (12 o’clock).

The circle of fifths can also be used to quickly determine the number of sharps or flats in a specified key (rather than superimposing the major scale pattern over the list of 12 note names). The key of C has no sharps or flats. As you move to the right from C (i.e., clockwise), each key has one more sharp. That is, the key of G has one sharp (F#), and the key of D has two sharps (F# C#). The additional note to be sharped is the preceding note of the circle (moving clockwise, i.e., FCGDAE). As you move to the left from C (i.e., counterclockwise), each key has one more flat. That is, the key of F has one flat (Bf), and the key of Bf has two flats (Bf Ef). The additional note to be flatted is the next note of the circle (moving counterclockwise, i.e., BfEfAfDfGfCf).

Do not confuse the procedure for forming chords with the procedure for finding related chords. A major chord is defined as any tone plus the tones four tones above it and seven tones above it on the 12-tone scale (e.g., 0-4-7 or 4-8-11). Using modern music terminology, it is a “root” note, a major third above it, and a fifth above it (e.g., AC#E or CEG). The related chords are the key chord, and the chords whose roots are five tones and seven tones above that. Or, in modern music terminology, the key chord plus the chord whose root is a fourth above that, plus the chord whose root is a fifth above that, i.e., I-IV-IV.

It is not the purpose of this article to reproduce the theory of modern music. It is, rather, to present information about the physics of music to assist an understanding of why music is the way it is. Most books on music simply teach the “theory” of music as a rote set of rules, without any physical explanation of why a twelve-tone scale is used, or why chords are defined the way they are, or why three chords whose roots are a “fifth” apart will “cover” a song, or why a major scale is used for so many songs.

For more information about modern music terminology, you can purchase a book on “music theory” or a good basic guitar (or piano) book in a music store. If you just want to learn to strum the guitar, without reading music, there is no need to do this – just read on. If you wish to learn to read modern musical notation, you will have to purchase a book, or take lessons.

5. Minor Chords in Major Keys

For a particular key, we have seen that there are three closely harmonically related chords based on this key – the tonic, subdominant, and dominant, or I, IV, V. But songs do not include just these three chords – they involve many others. For each of the seven notes of a key, there is a chord, and any of them may appear in a musical piece based on that key. These other chords are determined simply by writing down the notes of the key, selecting every possible selection of three alternating notes, and naming them according to the root note and what type of chord they define (major, minor, diminished, augmented).

For example, let us select the key of C (major). The notes of this scale are determined by selecting the notes from the 12-tone chromatic scale, starting with C, and using the major scale pattern 2212221:

Chromatic scale: A A# B C C# D D# E F F# G G# A
Or (starting with C): C C# D D# E F F# G G# A A# B C
(Diatonic) Scale for key of C major: C D E F G A B (step pattern 2212221)
Chords:
CEG. These are the notes 1,5,8 of the chromatic scale; or, in modern music notation, 2 steps, 1.5 steps, or root, major third, fifth; therefore the chord is Cmaj.
DFA, notes 1,4,8; or 1.5 steps, 2 steps, or root, minor third, fifth; therefore chord is Dmin
EGB, notes 1,4,8; or 1.5 steps, 2 steps, or root, minor third, fifth; therefore chord is Emin.
FAC, notes 1,5,8; or 2 steps, 1.5 steps, or root, major third, fifth; therefore chord is Fmaj.
GBD, notes 1,5,8; or 2 steps, 1.5 steps, or root, major third, fifth; therefore chord is Gmaj.
ACE, notes 1,4,8; or 1.5 steps, 2 steps, or root, minor third, fifth; therefore chord is Amin.
BDF, notes 1,4,7; or 1.5 steps, 1.5 steps, or root, minor third, diminished fifth; therefore chord is Bdim.

In summary, the chords of the key of C major are Cmaj, Dmin, Emin, Fmaj, Gmaj, Amin, Bdim, or (since the suffix “maj” is usually dropped, m is used for minor, + or aug is used for aug, and o or dim is used for diminished), C, Dm, Em, F, G, Am, Bo. For the major keys commonly used on the guitar, the list of all of the basic chords are as follows:

A: A, Bm, C#m, D, E, F#m, G#o
C: C, Dm, Em, F, G, Am, Bo
D: D, Em, F#m, G, A, Bm, C#o
E: E, F#m, G#m, A, B, C#m, D#o
G: G, Am, Bm, C, D, Em, F#o.

Note that it the chords of a key are listed in alphabetic order, then the major/minor sequence is always the same, viz., major, minor, minor, major, major, minor, diminished.

The material just presented (for determining which chords may be used in a particular key) is *very* important, and will be repeated. First, to determine the notes of a particular key, list all the twelve notes of the chromatic scale (A A# B C C# D D# E F F# G G#). To identify the notes of any (major) scale, select the first note (root, key note, name note) of the scale, and then the remaining notes according to the major scale pattern, 2212221. For example, the key of A consists of A B C D E F# G. The triad chords of the key are then determined by selecting all seven sets of three consecutive-alternating notes of the scale. In this (key of A) example, these are ACE, BDF#, CEG, DF#A, EGB, F#AC and GBD. These triad chords are major, minor, or diminished, depending on whether their step pattern is a major, minor, or diminished step pattern. (Hence the chords just listed are A(major), Bminor, Cminor, D(major), E(major), F#minor, and Gdiminished.) This, however, does not have to be checked. The sequence is always major, minor, minor, major, major, diminished (i.e., chord I is a major chord, II is minor, III is minor, IV is major, V is major, VI is minor and VII is diminished). Hence, for most popular songs, which are written in a major key, the chords involved are the major chords of the key, I-IV-V , which in this example are A-D-E. If it is desired to add (or substitute) another chord to a piece, it must be one of the chords of the key. That is it must be II, III, or VI (all minor chords) or VII (a diminished chord) – in the example, a Bm, Cm, F#m or Go. It is certainly possible for a "special effect" to include a chord that is not in the key, but this would be unusual.

When not referred to by their alphabetic names (A, A# etc.), the notes of a scale are called the "degrees" of the scale. For example, in the scale of C major (CDEFGABC), C is the first degree, D is the second degree, etc. The degrees of a scale have names: tonic, supertonic, mediant, subdominant, dominant, submediant, leading note, and tonic. The eight-tone major and minor "key-based" scales of Western music are called "diatonic" scales. The etymology of the word "diatonic" is Greek: diatonikos, stretched through (the notes of the full, chromatic, scale), from dia-, through + teinein, to stretch.

All of the chords in a particular key are collectively referred to as "diatonic" chords. All of the triad chords (root + third + fifth) of a scale (i.e., diatonic triads) are named after the degrees of the scale. These chords are also referred to by Roman numerals, I, II, III, etc. The sequence of diatonic chords in a scale are hence as follows:

I (tonic) – major
II (supertonic) – minor
III (mediant) – minor
IV (subdominant) – major
V (dominant) – major
VI (submediant) – minor
VII (leading note) – diminished.

The VII (diminished) chord is rarely used.

Diatonic Ratios. As noted above, the selection of seven tones of the 12-tone chromatic scale (e.g., a major scale pattern) is called a “diatonic” scale. It is interesting to examine, for the chromatic and diatonic scales, the values of the ratios of the frequencies of each tone to the frequency of the first tone of the scale. The ratios of successive tones of the 12-tone chromatic scale are shown in the table below. They are simply successive powers of the twelfth root of 2, $2^{1/12} = 1.059463094$. The major (diatonic) scale uses tones 0, 2, 4, 5, 7, 9, 11, and 12 of the chromatic scale (major scale pattern 02212221). The ancient Greeks admired symmetry, and they tried to find simple fractions that were close to the values of the frequency ratios. The closest fractions that they found are shown in the table. These ratios are called “diatonic ratios.” The rather remarkable thing is that the values of the frequency ratios for the diatonic scale can be approximated very closely by very simple fractions.

Ratios of the Frequency of Each Scale Tone to the Frequency of the First Scale Tone				
Chromatic Scale		Diatonic Scale		Diatonic Ratio
Number	Ratio	Number	Ratio	
0	1	0		0
1	1.059463094			
2	1.122462048	2	1.122462048	9/8=1.125
3	1.189207115			
4	1.25992105	4	1.25992105	5/4=1.25
5	1.334839854	5	1.334839854	4/3=1.33333
6	1.414213562			
7	1.498307077	7	1.498307077	3/2=1.5
8	1.587401052			
9	1.681792831	9	1.681792831	5/3=1.66667
10	1.781797435			
11	1.887748625	11	1.887748625	15/8=1.875
12	2	12	2	2

Summary. In popular songs, the most important chords are the tonic (I) and dominant (V) chords. A large number of songs involve just these two chords. The dominant chord often has a fourth note (a (minor, or flatted) “seventh” interval) added as a lead-in back to the tonic chord. The next most important chord is the subdominant chord (IV). Chord progressions are sequences of chords from the tonic chord to other chords in the key (i.e., diatonic chords), and back to the tonic chord. A very large number of popular songs involve just the three chords I-IV-V.

To play a full range of songs, you will have to know minor chords. For example, the song, “Morning Has Broken,” in the key of C, involves the chords C, G, F, Am, Dm, Em, D7, and G7 – every major and minor chord in the key of C, plus two seventh chords! Similarly, "The House of the Rising Sun" contains several minor chords.

6. Chord Substitutions

For variety in a piece of music, it may sound good to substitute one diatonic triad chord with another from the same key that differs in only one note (i.e., that shares two notes). You may verify that diatonic triads (three-note chords of a given key) that are separated by a third (i.e., the chord names are two letters or Roman numerals apart) share two notes. In Roman-numeral symbology:

Chord I shares two notes with chords VI or III;
Chord IV shares two notes with chords II or VI;
Chord V shards two notes with chords III or VII.

For example, in the key of C:

Chord C shares two notes with chords Am or Em;
Chord F shares two notes with chords Dm or Am;
Chord G shares two notes with chords Em or Bo.

Or, in the key of G:

Chord G shares two notes with chords Em or Bm;
Chord C shares two notes with chords Am or Em;
Chord D shares two notes with chords Bm or F#o.

The II chord and the IV chord are often be substituted for each other, as in the sequences I-IV-V-I and I-II-V7-I. For example, the main verse of the song “Jamaica Farewell” uses the sequence I-IV-V-I, and the chorus uses I-II-V7-I. The song sounds all right if the former sequence is used for both the verse and the chorus, but it does not have as much variety. In the key of C, these sequences are C-F-G-C and C-Dm-G7-C. In the key of G, these sequences are G-C-D-G and G-Am-D7-G. In the key of A, we have A-D-E-A and A-Bm-E7-A. The use of the minor chord in place of the major one gives the verse a “wistful” sound.

The seventh may be dropped at times, for additional variety. In making substitutions, one must be guided by one's ear. For example, in "Jamaica Farewell," the sequence G-Am-D7-G sounds fine, but G-Em-D7-G does not. Note that a frequently used progression is from the dominant chord to the tonic chord (a fifth apart), e.g., from E to A or B to E. Hence, the progression A-Bm-E7-A in "Jamaica Farewell" sounds particularly pleasant (two successive dominant-tonic progressions).

7. Minor Keys

The keys listed above are all major keys, i.e., they are based on the major scale pattern 2212221. This section will address keys based on minor scale patterns. Unlike the major scale, where there is a single scale pattern, there are several different minor scale patterns. The three standard ones are the ascending melodic minor scale, the descending melodic minor scale, and the harmonic minor scale. (A piece written in a minor key may include notes from all three minor scales.) The first two of these are used in the melody, and the third is used for the harmony, i.e., for constructing chords. The term “ascending” refers to going up the scale (ABC…), and the term “descending” refers to going down the scale (GFE…). The ascending melodic minor scale is defined by the tone interval pattern 2122221. The descending melodic minor scale is defined by the *descending* pattern (from the top note on the scale) 2212212, or the *ascending* pattern 2122122 (from the bottom note of the scale). The harmonic minor scale is defined by the (ascending) interval pattern 2122131, i.e., it has an unsharped sixth degree (scale note) and a sharped seventh degree. Note that the first four intervals are the same for all three minor scales (2122).

Why are there three (standard) minor scales? Well, melodies in minor scales tend to use sharped sixth and seventh degrees (notes of the scale) when they are going up, but unsharped notes when they are going down. With respect to chords, the sharping of the seventh degree changes the V chord from a minor to a major. The key then has two minor chords and four major chords, instead of three of each. This is a cultural preference in Western music – having three minor chords in a piece sounds rather gloomy, or heavy, or oppressive.

The descending melodic minor scale is the one used to define the key signature, and the harmonic minor scale is the one used to define chords (on the descending melodic minor scale). The key signature that is used for a minor key is the major-key signature having the same number of sharps or flats as the descending melodic minor scale. Hence, for each major scale there is a corresponding minor scale, referred to as a "relative" minor scale. (Note that although major and minor keys that share the same key signature are called relatives, the "relation" is simply in the key-signature notation, not in the music. In key-based music, it is the tonic note that defines the scale. Two scales with the same key note (e.g., A major and A minor) are closely related; a major key and a relative minor key are not. Later, we shall discuss another relative minor key.)

The correspondence between these is as follows:

Number of sharps or flats	Major key	Relative minor key
None	C major	A minor
1 sharp (F)	G major	E minor
2 sharps (FC)	D major	B minor
3 sharps (FCG)	A major	F# minor
4 sharps (FCGD)	E major	C# minor
5 sharps (FCGDA)	B major	G# minor
6 sharps (FCGDAE)	F# major	D# minor
1 flat (B)	F major	D minor
2 flats (BE)	Bf major	G minor
3 flats (BEA)	Ef major	C minor
4 flats (BEAD)	Af major	F minor
5 flats (BEADG)	Df major	Bf minor
6 flats (BEADGC)	Gf major	Ef minor

For example, let us select the key of A minor. The ascending melodic A-minor scale is ABCDEF#G#A. The descending melodic A-minor scale is AGFEDCBA. This is the same selection of notes as for the key of C major, so the key of A minor is the relative minor key of the key of C major. The harmonic A-minor scale is ABCDEFG#A. The notes of this (descending melodic minor) scale are determined by selecting the notes from the 12-tone chromatic scale, starting with A, and using the minor scale pattern 2122122:

Chromatic scale: A A# B C C# D D# E F F# G G# A
Scale for key of A minor: A B C D E F G A (step pattern 2122122). This is the same scale as the key of C major (no sharps or flats).
Chords:
Remember: The chords have a sharped seventh degree, or G#.
ACE. These are the notes 1,4,8 of the chromatic scale; or, in modern music notation, 1.5 steps, 2 steps, or root, minor third, fifth; therefore the chord is Amin.
BDF, notes 1,4,7; or 1.5 steps, 1.5 steps, or root, minor third, diminished fifth; therefore chord is Bdim.
CEG, notes 1,5.8; or 2 steps, 1.5 steps, or root, major third, fifth; therefore chord is Cmaj.
DFA, notes 1,4,8; or 1.5 steps, 2 steps, or root, minor third, fifth; therefore chord is Dmin
EG#B, notes 1,5,8; or 2 steps, 1.5 steps, or root, major third, fifth; therefore chord is Emaj. (Note: without the sharped seventh degree, this chord would have been Emin.)
FAC, notes 1,5,8; or 2 steps, 1.5 steps, or root, major third, fifth; therefore chord is Fmaj.
GBD, notes 1,5,8; or 2 steps, 1.5 steps, or root, major third, fifth; therefore chord is Gmaj.

Note that if it were not for the sharped seventh degree, the chords of the A-minor key would have been identical to those of the C-major key. This is obvious since the notes of these two keys are identical. By sharping the seventh degree, the Em is changed to E.

In summary, the chords of the key of A minor are Am, Bo, C, Dm, E, F, G. For the minor keys commonly used on the guitar, the list of all of the basic (triad) chords are as follows:

A minor: Am, Bo, C, Dm, E, F, G
B minor: Bm, C#o, D, Em, F#, G, A
C# minor: C#m, D#o, E, F#m, G#, A, B
D minor: Dm, Eo, F, Gm, A, Bf, C
E minor: Em, F#o, G, Am, B, C, D
F# minor: F#m, G#o, A, Bm, C, D#, E.

Note that the major/minor sequence is always the same: major, diminished, major, minor, major, major, major. The diatonic descriptors (tonic, dominant, etc.) are the same for the minor scales as the major scale.

For each of the preceding keys, the effect of sharping the seventh degree is to change the V chord from a minor to a major. Leaving it as a minor sounds all right – it just depends on the song. All minor keys use the sequence 2122 for the first five degrees, but the selection of tones and chords varies somewhat beyond that. The selection of chords in minor-key songs is not ironclad. For example, the song “The House of the Rising Sun” uses the chord sequence Am-C-D-F-Am-C-E-E7-Am-C-D-F-Am-E-Am-E7. The chords of the key of A minor are Am, Bo, C, Dm, E, F, and G. This key uses the chord Dm, whereas the song uses D. The song sounds all right if Dm is used, but it is not the song as it is usually played. It sounds “brighter” with a D in place of a Dm.

8. Other Scales

The most common scales are the major and minor scales described above. Another (minor) scale you may encounter is the Dorian mode of a major scale. In ancient Greece, the major scale was called the Ionian mode. The scale starting from the second degree of the major scale was called the Dorian mode. Similarly, the scales starting from each of the other degrees had names (Phrygian, Lydian, Mixolydian, Aeolian, and Locrian). The major scale pattern is 2212221, so the Dorian mode (scale) pattern is 2122212. The Dorian mode is a minor scale, since it has a flatted third (or since the scale pattern starts as 2122). It differs from the minor scale described above, however, since it has a natural sixth degree and a flatted seventh degree. Note that the descending melodic minor scale discussed earlier is the mode starting on the sixth degree of the major scale.

Since the notes of the Dorian mode are exactly the same as the major (Ionian) scale, the chords are exactly the same, as well. The only difference is that the tonic note (and chord) is one degree higher.

“Scarborough Fair” is an example of a song written in the Dorian mode. Suppose that it is played in the Dorian mode of the key of C. Then the notes are:

DDAEFED
ACDCABGA
DDDCAAGFE
DAGFEDECD,

and the chords are:

Dm-C-Dm
…G-Dm
…F-C
Dm-C-Dm.

Note that all of the notes, and all of the chords, are from the key of C. The song is written in the Dorian mode of the key of C major. Most songbooks would indicate that the song is written in the key of D minor (relative minor of the key of F). But the D minor scale has a B flat and a Gm chord, whereas the song has B natural and a G chord. It is hence simpler to notate this song using the C key signature, with the tonic note (the note that starts and ends the song) as D. (If it is written in the key of D minor, then the relative major key is F, which contains a B flat. Since the song contains no B flat, it would be necessary to cancel the B flat with an accidental (natural sign). Further confusion arises since the key of D minor includes the chord of Gm, not G. Notating the song as the Dorian mode of C requires no changes at all to either notes or chords.) Since the Dorian mode of the C major scale uses the same key signature as C major, it is a relative minor key of C major (just as the key of A minor discussed earlier was a relative minor key of C major).

9. The Blues Scale

The “blues scale” usually involves just five notes – 0, 3, 5, 7, and 10 of the 12-tone scale. For example, A, C, D, E, G, or G, Bf, C, D, and F. It uses primarily the dominant seventh chords I7, IV7, V7, e.g., G7, C7, D7, or Bf7, Ef7, F7, or A7, D7, E7, or Amin7, Dmin7, Emin7. The following is a table that shows the blues scales and the corresponding major scales from which they are extracted.

Blues Scale	Major Key	Diatonic Chords
G	Bf	Bf Cm Dm Ef F Gm Ao
G#(Af)	B	B C#m D#m E F# G#m A#o
A	C	C Dm Em F G Am Bo
Bf	Df	Df Efm Fm Gf Af Bfm Co
B	D	D Em F#m G A Bm C#o
C	Ef	Ef Fm Gm Af Bf Cm Do
C#	E	E F#m G#m A B C#m D#o
D	F	F Gm Am Bf C Dm Eo
Ef	Gf	Gf Afm Bfm Cf Dfm Efm Fo
E	G	G Am Bm C D Em F#o
F	Af	Af Bfm Cm Df Ef Fm Go
F#	A	A Bm C#m D E F#m G#o

The “blues” scales and chords are used in lead and rhythm guitar playing, as background to rock and other music. If you are interested in learning to play this style of music, you will surely buy a book on the method. The material presented in this article will help you understand the theory behind keys and scales, but it does not discuss the technique of the blues style.

IV. How to Play the Guitar by Ear

A. Objectives

At this point, you know what chords (in sets of three) are harmonically related (or can quickly determine them). Since a very large proportion of popular songs is based on just three related chords (a fifth above and below the root chord), you know the chords for a lot of songs. So what else do you need to know, or know how to do? The following is a list of what else you need to determine, learn or accomplish.

1. How many chords should you learn?
2. What types of chords should you learn?
3. How to finger the chords
4. How to strum
5. How to determine what chord to play at each point of a song
6. How to determine what key to use
7. How to sing and strum at the same time.

We shall address each of these items in order.

I am not going to discuss timing (e.g., 3/4 time versus 4/4 time) – that is discussed in any first book on the guitar. It helps if you move your toe to the “beat” of the music. Do not tap your foot up and down – that is distracting to many people. Also, I am not going to discuss particular methods of play, such as blues or jazz. This article is just about the basics of accompanying popular songs by ear.

B. How Many Chords Should You Learn?

It is not necessary to learn to play in all of the twelve different keys (twenty-four if you count relative minor keys). Most songs accompanied with a guitar are written in just a few keys. The most popular keys are A, C, D, E, and G. These keys cover the octave well, and so it is possible to select one or more of them for which you will, with your vocal range, be able to sing a particular song comfortably. There are several reasons why these keys are popular.

First, they cover the octave well (i.e., completely and at a reasonable resolution), so there is no need for more chords (to match anyone's vocal range). Second, for the guitar they contain a large number of "full" and easily fingered and easily strummed chords. By the term "full" I mean a chord that includes all six guitar strings. "Easily fingered" means that usually just three fingers are required (in "home" position), and that the fingering is not awkward. "Easily strummed" means that (since the E and A (bass) strings are included) the "alternate bass" strum pattern (described later) can be used without fear of inadvertently hitting a note that is not in the chord. Finally, learning chords requires some effort, and there is no point to learning a vast number of them when a smaller number will do just as well. From time to time you will have to learn another chord to do a particular song, and you can learn such chords when the need arises.

The "basic" (harmonically related) chords for the keys listed above are as follows.

Key of A: Chords A D E
Key of C: Chords C F G
Key of D: Chords D G A
Key of E: Chords E A B
Key of G: Chords G C D.

The preceding keys correspond to the segment C G D A E of the circle of fifths. These keys contain zero, one, two, three and four sharps. By tradition, music for instruments that are played on the treble clef (e.g., guitar, trumpet, clarinet) is usually written in keys having sharps, and music for instruments that are played on the bass clef (e.g., trombone, baritone) is usually written in keys having flats. The restriction of the keys to the set C G D A E is traditional for the guitar.

The basic chords for the keys C G D A E correspond to the full "natural-name" (unflatted and unsharped names) segment F C G D A E B of the circle.

You should learn all of the "natural-name" chords, A B C D E F G, since these are the basic chords of the five popular keys that span the octave (C G D A E). These are "major" chords. In addition, you should learn all the "minor" natural-name chords, Am Bm Cm Dm Em Fm Gm, and all of the "seventh" chords, A7 B7 C7 D7 E7 F7 G7. Any other chords that you need you can learn as you go along.

Learn these chords well, so that you can play them easily from memory, and without looking at your fingers. When you start, your fingers will get sore, even with nylon strings. In a couple of weeks, your fingertips will develop calluses, and no longer be sore.

There are many other chords than the ones (major, minor, and seventh chords) mentioned above, and they are used in many pieces. As you continue, you may wish to learn and use them. They add variety and richness to music, but you can accompany a large number (and proportion) of popular songs without them. To get well on the road to playing by ear, the ones listed above will do just fine for starters.

C. What Types of Chords Should You Learn

There are basically two types of chords – barred (or barre or bar) chords, and unbarred chords. A barred chord is one in which your index finger presses some (partial barre or small barre or half barre) or all (grande barre or full barre or large barre) of the strings along a fret, and the other three fingers press selected strings at higher frets. You can now play other chords all the way up the fretboard simply moving your index finger to a new fret and keeping the same chord "shape" with your other fingers. Your index finger is serving as a movable "nut" or capo.

Barre chords are a little more trouble to learn than unbarred ones, but they give you tremendous flexibility in playing chords all the way up the fretboard, using the same chord "shape." If you want to learn to play the electric solidbody guitar, you will definitely want to learn barre chords, because you will be playing the full range of the fretboard all the time. If you want to learn classical guitar or flamenco, you will need to know barre chords. Barre chords are harder to play on a steel-string acoustic guitar than on a nylon-string guitar. If you just want to "strum along" to accompany popular music, then you do not need barre chords.

The only real exception is the F chord. If you play this unbarred, you play only the first four strings. If you play it barred, you play all six strings – a much fuller, richer-sounding chord. Also, when strumming, there is a chance with the shorter chord of hitting one of the top two (bass) strings. (Note: The strings on a guitar are numbered 1 through 6, starting with the *thinnest*, i.e., E G B D A E. The first four strings are hence E A D B. This is the reverse of the procedure for naming notes, which runs from lower frequencies to higher frequencies, i.e., A B C….)

There are two basic barre chords that cover all six strings. These are based on the E chord and the A chord. The F chord, for example, is the same as an E chord, but with the index finger barring the first fret. The A# chord is just an A chord, with the finger barring the first fret. You can barre other chords, such as the C chord and D chord, but you will just use the first four or five strings, not all six.

If you wish to play a six-string C chord, play it with the ring and little fingers on the third fret of the last two strings (E and G). It sounds better than playing the E string open.

If you are going to learn to play barre chords, it is best to make that decision at the start. The reason for this is that if you play barre chords, you usually want to play chords in the open position with the fingers other than your index finger, if you can. It is then free to form a barre at a higher position, without changing the “shape” of your fingers. For example, if you play an A with the middle, ring, and little fingers, then you can move instantly to an A# or B by sliding these fingers up the fretboard and depressing your index finger to form the barre. If you were to play the A using your index finger, you would have to change the position of all of your fingers when you moved to the barred chord.

Another reason for making the decision at the beginning is that if you play barre chords, it is necessary for the thumb to be placed behind the neck of the guitar (so that you will be able to press all the strings down behind a fret with your index finger). If you let your thumb wrap around the neck, you will not be able to play a barre chord without completely readjusting the position of your left hand. Some people prefer to wrap the thumb around the neck to mute the first bass string when it is omitted from a chord based on (the top) four or five strings, e.g., D, so that it does not sound if accidentally struck in a strum. My personal opinion is that the thumb should always be placed behind the neck, but when I am tired or strumming loud and fast I will wrap the thumb around the bass E string to mute it.

The big advantage of knowing how to play barre chords is that you can play a vast number of chords just from knowledge of the basic E and A chords. You can play an A, Am, A7 and Am7 easily with your middle, ring, and little fingers. Hence, by moving up the fretboard with your index finger barring the frets, you can also play A#, A#m, A#7, and A#m7; B, Bm, B7, Bm7; and so on. You can play an E, Em, E7, and Em7 easily with your middle, ring, and little fingers. Hence, barring the first fret, you can play F, Fm, F7, and Fm7; F#, F#m, F#7, and F#m7; and so on. The C chord shape can also be used as the basis for barre chords, but they are not six-string chords.

Another advantage of knowing barre chords is that you can play songs in a wider variety of keys, without having to learn a lot of new chords. For example, some popular songs are recorded in the key of Bflat. If you want to accompany this song on a recording, or play it in this key, you will have to use a capo or use barre chords. (You do not *have to* use barre chords, but it is simpler to use them than to learn the new chords of the key of Bflat.) You can accompany such songs without using a capo if you know barre chords, but it is much easier to play in an unfamiliar key (e.g., Bf) by using a capo and playing in a familiar key (e.g., A).

There is nothing wrong with using a capo, and it is often very advantageous, but there is a drawback. First, you have to use it for the entire song, and use it for all chords, even if you need it only for one or two. When you put the capo on, you are denied the opportunity of using the full strings (or any frets above the capo), which are richer in tone than those higher on the fretboard. You have hence muted the quality of the song somewhat (for all chords that could have been played above the capo).

D. Using a Capo

The main reason for using a capo is to accompany other instruments for which music is customarily written in keys not ordinarily used by guitarists. Most guitarists will play a song in one of the keys C, G, D, A or E. The choice depends mainly on the key that best matches the guitarist's vocal range, but some songs simply "sound better" in certain keys (because of the way in which the chords are formed). The problem that arises is that players of other instruments often use other keys. For example, a clarinetist or trombonist will usually be more familiar with the keys F, Bf, Ef, Af and Df. (The keys C, G, D, A and E have no sharps or flats, one sharp, two sharps, three sharps, and four sharps, respectively, and the keys F, Bf, Ef, Af and Df have one flat, two flats, three flats, four flats, and five flats, respectively.) If a guitarist and a clarinetist wish to play together by ear, one of them will be forced to play in unfamiliar keys.

This problem is easily resolved by using a capo, placed on the first fret of the guitar. This has the effect of raising the key by one "semitone," i.e., sharping or flatting each note. If the guitarist usually plays a piece in the key of A, placing a capo on the first fret will raise the key to A#, or Bf (keys are usually referred to as natural or flat keys, rather than as sharp keys). With the capo on fret one, he may play (finger) the song as he usually does in the key of A, but the music will be in the key of Bf, and the clarinetist may easily accompany the key of Bf.

The capo changes the (fingered) keys C, G, D, A and E to the (voiced) keys Df, Af, Ef, Bf and F, respectively. The guitarist may readily accommodate the clarinetist or trombonist (or any other instrument that customarily uses "flat" keys) by placing the capo on fret one. The only keys missing from the combination of these two lists are the keys of Gf (six flats, or F#, six sharps) and B (five sharps, or Cf, seven flats). Practically no pieces are written in these keys, and so there is virtually no occasion to warrant placing the capo on the second fret, assuming that the guitarist is familiar with all five keys, C, G, D, A, and E. (If the guitarist wished to change a key by two semitones, then he would place the capo on fret two. But this would simply change the keys C, G, D, A and E to D, A, E, B and G, and not help the problem of scale accommodation a whit.)

E. How to Finger the Chords

For illustrations of how to finger chords (barre or not), see any book on how to play the guitar, or a “chord chart” (which may be a little book, not a chart at all). Many guitar books include a chord chart at the back.

F. How to Strum

This article is about strumming by ear, not about picking. If you are serious about picking, you will probably need to learn to read music. There are several methods of picking, and you may want to learn them. To use them effectively, you will probably have to learn to read music, either standard music notation or tablature.

There are a variety of ways to strum, and I recommend that you learn a simple method first, followed by more complex methods. The simplest method is simply to strum across all of the notes of the chord with your thumb, one strum per beat. Take care not to include strings that are not part of the chord, e.g., when strumming a D, you must omit the sixth string (low E). You may even omit the fifth string (A), although it is part of the D chord, since it is slightly preferable to have the root note of the chord as the first one struck. (Similarly, for a C chord, you may omit the sixth string, which is either E or G, depending on how you finger the chord). The drawback of this method is that it is monotonous. The advantage of this approach while you are learning is that you hear the chord very distinctly, and you can more easily match the chord to the melody.

Another method of strumming is to pluck either of the bass strings of the chord with your thumb (in a downward motion), and then pluck the three or four treble strings (simultaneously) with your fingers (pulling them toward you). You pluck the bass strings alternately (with the finger-pluck in between). It sounds better if first string plucked with the thumb is the root of the chord. For example, when plucking the D chord, pluck D with the thumb, followed by the three treble strings with the fingers, followed by A with the thumb, followed by the three treble strings with the fingers. The advantage of this method is that you have lot of control over the strings. You will rarely miss a string, because your hand does not move across the strings. It should perhaps be referred to as “plucking” or “picking” rather than strumming, but you are plucking several strings simultaneously, not in arpeggio style. This method works well for waltzes (3/4 time).

A disadvantage of this method for beginners is that it is harder to tell whether you are sounding the right chord. It is much easier to “know” if a chord is correct if you strum all of the chord notes in one motion. If you strike a bass note and then several others, it is more difficult to sense that the chord is the correct one.

Another method of strumming is to pluck a bass string with the thumb, and then sweep the fingers over the notes in a downward brushing motion. The bass strings are alternated (hence the term “alternate-bass (or alternating-bass) strum”). The disadvantage of this method is that you can do about half as many strums per unit of time as with most other methods.

Should you use a pick, or plectrum? The answer depends on what type of guitar you want to play, and how loud you want the sound to be. It also depends on how elaborate you want the strum patterns to be. If you want to play a solidbody electric guitar, you will use a pick. If you play an acoustic, you may or may not use a pick. If you want to play loud, use a pick. If you want to play elaborate or fast strum patterns, use a pick. In general, the answer is yes, do learn to play with a pick. It is easier to learn to play without the pick, but you should master it as soon as you know the basics (strumming with the fingers to the accompaniment of singing).

Even if you learn to play with the pick, you may not use it for all songs. If you play a classical flamenco piece such as Malagueña on a classical guitar, you will not use a pick.

The “pick” that I am referring to is the standard, “heart-shaped,” flat pick. In addition, there are thumb picks and finger picks. These are used in different types of music. I will not address the use of these picks here.

With the pick, you may strike the strings on either the downward sweep or the upward sweep. By not striking the strings on selected sweeps, either downward or upward, you can produce a variety of strum patterns.

With the pick, you may alternate the picking of bass strings. If the song has a slow or moderate tempo, use down strokes for the bass strings and for the strums of the treble strings. If the tempo is fast, use down strokes for the bass strings and up strokes for the strums across the treble strings. (That is, strike the first bass string on a downward strum, catch most of the treble strings on the upward strum, then strike the second bass string on a downward strum, and catch most of the treble strings on the upward strum.)

There are a variety of strum patterns, and these are discussed in many first books on the guitar. If D denotes a downward sweep of the pick and U denotes an upward sweep, then a "calypso" strum pattern is DDUUD. Another pretty pattern is DDUUDU. The strum pattern must match the "beat" of the song. I am not spending much time on this topic, because it is not difficult to master. You can spend as much time on it as you wish, after you learn to play by ear.

G. How To Determine What Chord to Play at Each Point of a Song (Chord Progressions)

Knowing what chord to play at each point of the melody is the essence of learning to play by ear. And it is the most difficult part to learn (and to teach). Here is how to do it.

For starters, pick a simple, popular song, that has a regular, repeated pattern. For example, "The Wreck of the Old 97," or "You Are My Sunshine." I will assume that you do not have a recording of the song, so you can play the piece in any key. The key of A is very popular for guitar pieces – chords A, D, and E. Start out by strumming the A chord with your thumb, in repeated downward strokes. While you are doing this, hum the melody of the song (or whistle it, if you are a whistler). It is simpler if you do not try to sing the song at this point, since that makes it too complicated. Similarly, do not try to use a pick or elaborate strum pattern. All we want to accomplish at this point is match the chord to the melody. This is most easily done with the full downward strum of the thumb over the strings, while you are humming. The process has been simplified to the bare minimum. All you have to do is hum and strum. There is the least to keep track of, and the chord is as complete as it can be. If you cannot determine how to hum the song to accompany the chord that you are strumming, then I cannot help you. (This means that you probably cannot even sing along with a group. If you cannot carry a tune at all, e.g., sing in the shower or accompany a song on the radio or in church, then you are in trouble. On the other hand, if you are that limited in musical ability, you are probably not that interested in music, either, and are probably not reading this article.)

Now, as you are humming the tune while strumming, you will eventually come to a point at which the melody is no longer in tune with the chord that you are strumming. Now the big problem has arrived! The chord that you were strumming is no longer in tune with the melody, and you must switch to a different chord. Fortunately, for most simple songs, there are just two possibilities. Since you are playing the song in the key of A, and you started with the A chord, the next chord must be a D or an E (remember A B C D E, 1-4-5, A-D-E). (If it is not, then you did not pick a simple melody!) You will sense that the chord following the A is higher or lower, but that does not help a lot, since the chords are on an endless circle (i.e., D and E are both below and above A). So what do you do?

I assume that you, as I, do not have an incredible ear for music, so that you do not automatically know whether the next chord is a D or an E. So what you do is try both possibilities, and pick the one that sounds right. Believe me, one will usually sound right, and the other one will usually sound wrong. Let us suppose that the next chord is a D. Continue strumming and humming until you come to another point at which the chord no longer sounds correct. At this point, there are two chords that you can move to: back to A, or on to E. Since you were just playing an A, you may have a strong sense that the new chord is A, and move back there. If you do not, the next chord is most likely E. Try it and see. Whichever chord worked, continue with it until it no longer matches the melody. Let us assume that it was the E. When that chord no longer "works," you will probably move back to the A.

Why? Because (as discussed earlier) most songs consist of segments that start with the tonic chord, move on to a few other chords, and return to the tonic chord. Patterns such as this occur over and over, until the song is over, ending on the root chord. As mentioned earlier, these "patterns" are called "chord progressions." In the example just given, the chord progression was ADEA. That is a very popular chord progression, which occurs in many songs.

In many cases, if the next chord in a song is of lower pitch than the tonic chord (e.g., A), it will be the dominant (higher) chord (E), and if the next chord is higher than the tonic chord it will be the subdominant (lower) chord (D). The reason for this is that dominant chord (E) is the closer chord when moving down from the tonic chord (A), and the subdominant chord (D) is the closer chord when moving up from the tonic chord.

Another thing you will notice is that in many songs, particularly simple traditional melodies, each successive chord is played for the same duration (i.e., is played for the same number of beats or measures). This is because many songs are in fact poems with a fixed number of syllables per line, and a particular chord often accompanies each line.

In any event, you have now identified the chords that accompany the melody. Repeat the song a few times. After a few times, you will have the chord change-points and progression memorized.

Now, let's change keys. Let's do the same song, but in another key. The process of moving to another key is called transposition. There are several ways you can transpose. Suppose that you were able to hum the melody reasonably well in the key of A, but you would like to try a slightly lower key. Then let us change to the key of G, which is two tones below A (AG#G).

One method of transposing is to move each note of the chord progression down two tones (one letter note): A down to G, D down to C, E down to D, and A down to G, so that the chord progression becomes G-C-D-G. This is not a good way to solve the problem. It is fine if you are going over a sheet of music (in the key of A) on which the chords were indicated, and want to change them all to the key of G. But reading the chords from a sheet of music will not help you play by ear, so it is a poor practice to follow. Moreover, it would be virtually impossible to transpose a piece that you knew in one key to another key, "on the fly," while you are performing, using this method.

A much better (practical) method is to understand the form of the chord progression that was just identified, and apply it to the new key. If you want to learn to play by ear in any key, this is the approach that you should use. In this case, the progression was A-D-E-A. Recall that in the set of three related chords, one is the tonic, one is seven tones (a "fifth") above the tonic, and one is seven tones (a "fifth") below the tonic. But a note that is seven tones below the tonic is the same as the note five tones above the tonic (i.e., an octave higher). In terms of modern musical terminology, a note that is a "fifth" below the tonic is the same as the note a "fourth" above the tonic (remember to include both the starting and ending notes when counting fourths and fifths).

If we denote the notes of a scale by numbers, then a major chord may be denoted by the notation root-3rd-5th (i.e., a major chord consists of the root note, the fourth above it (which is the fifth below it), and the fifth above it; for example, ADE). The terminology for naming chords is similar. The names of the notes of a scale are tonic, supertonic, mediant, subdominant, dominant, submediant, and leading note. They are referred to in Roman numerals as I, II, III, IV, V, VI, and VII. The chords that have each of these notes as roots are similarly designated. In this terminology, the chord progression A-D-E-A is I-IV-V-I. In the key of G, the progression is exactly the same, I-IV-V-I. In the key of G, the three related chords are GDE (remember, G A B C D E, hence I-IV-V is G-D-E). Hence the chord progression of the example is G-D-E-G. It is the chord progression in the Roman numeral terminology that is important and should be remembered.

The preceding may seem complicated, but after playing for a few weeks, and figuring out the chords of melodies, you will soon become very familiar with the progression patterns involved for each song. Play each song in several keys. You will instantly remember that the progression A(I)D(IV)E(V) is equivalent to C(I)F(IV)G(V), or to E(I)A(IV)B(V). You should try to remember these chord progressions in terms of the Roman numerals, as I-IV-V, rather than by the Roman letters, which are key-specific.

Terminology reminder (in case you skipped the theory section). The first note of the scale, or the key-note, is called the "tonic" note. The perfect fifth above the tonic is called the "dominant," and the perfect fifth below the tonic is called the "subdominant." The three basic chords that are closely harmonically related are the chords whose roots are the tonic, subdominant, and dominant chords, and they are denoted as chords I, IV, and V.

You will find that there is a tendency in chord progressions for the V chord (the dominant) to precede the I chord more than the IV chord (the subdominant) does. The reason for this is that the V chord contains the note just below the "tonic" note that defines the key, and it sounds very natural for that note to "lead up" to the tonic note, as the piece ends or as a section of the piece completes. It is for this reason that the note just below the tonic note is called the "leading note" of the scale. For example, the chord progression C-F-G-C sounds a little more natural than C-G-F-C, since the G chord contains the leading note B, but the F chord does not.

Something that will help you remember which note is the fifth (without counting) is to get in the practice of playing dominant seventh chords prior to returning to the tonic chord. The sequence G-C-D-G sounds fine, but the sequence G-C-D7-G, which includes a dominant seventh chord just prior to the return to the tonic chord, often sounds better. The reasons why this sequence sounds good were discussed earlier.

I-IV-V or I-IV-V7
C-F-G or C-F-G7
G-C-D or G-C-D7
D-G-A or D-G-A7
A-D-E or A-D-E7
E-A-B or E-A-B7.

In fact, B is difficult to play, so E-A-B is usually not used at all. Instead, E-A-B7 is the usual chord set (on the guitar) for the key of E.

After a while, you will automatically remember that in the chord set C-F-G, it is the G that is seventhed, not the F. Or, in the chord set D-G-A, it is the A that is seventhed, not the G. In general, you will become very aware of which chord is the IV (subdominant) chord, and which is the V (dominant) chord, in a set of three related chords (I-IV-V). For simple melodies (and this includes many popular songs), playing chords by ear is about as easy as playing a song with just three notes (chords I, IV, and V (with an occasional V7 for added flavor)).

If you follow the above method of changing keys, you do not have to remember anything at all about moving every note of a piece up or down by a fixed number of notes (i.e., transposing) to change the key. Unfortunately, when transposing from one key to another is discussed in music books, it is almost always the “move all the notes up or down by a fixed amount” approach that is described.

To get used to changing keys, take a particular song (e.g., “The Wreck of the Old 97”), and play it in every one of the keys that we are using: C G D A E. Do this over and over again, until you can play the piece in any key. You will find that you soon remember whether you are moving to chord IV or chord V, and have no problem doing this. At this point, when someone asks you to move to a lower or higher key, you will be able to do so with no difficulty at all, even if you have never played the song in the new key.

After you have mastered changing keys on a simple song that stays with one chord for a while, try more complicated songs, that change chords more frequently. For example, consider the first line of the song, “Ashes of Love.” If you play it is the key of A, you will play the words “Ashes of love” with the A chord, and then “cold” with D, “as” with A, and “ice” with E. Get used to changing chords quickly, and without looking at your hand.

You will encounter some songs in which the three major chords are not sufficient for the song. In some cases, one of these three chords may not sound quite right, but it may be used without causing too much of a problem. Your voice will sing the correct note, and the discord will not be noticeable. In other cases, you will simply have to determine what is the correct chord to play, or it will sound noticeably wrong. For example, in the song, “Jamaica Farewell” played in the key of A, the chorus (“But I’m sad to say, I’m on my way, Won’t be back for many a day…), sounds best if played as A-Bm-E7-A, but sounds all right if played as A-D-E-A (or A-D-E7-A). As another example, consider the song, “I Don’t Hurt Anymore,” played in the key of A. In the second verse of the song there is a line, “But now that I find (in D), You’re out of my mind (in D), I can’t believe that it’s true (in B7 then E)”, that must be played with a B7 chord. Accompanying it with an E or a D will not sound right.

As you build up a repertoire of songs, you will encounter more and more of these cases. For the "easy" cases such as exemplified above, you should be still able to change keys with ease, but for songs containing many chords other than the three basic ones, you will probably have to memorize the chord sequence in just one key.

Once you have mastered playing of chords and changing keys, it is much easier to accompany a recording. If you try to accompany recordings when you are first learning, you will find it somewhat difficult, for a number of reasons. First, your guitar must be in perfect tune. While playing for yourself, your guitar must be "in tune" (i.e., the notes correct relative to each other), but it does not matter what the starting note for the guitar is. It could be E329.6 (the correct frequency), or it could be any other frequency, as long as the other strings are tuned to that string (see any book on guitar to see how to tune the strings of a guitar relative to each other). If you wish to accompany a recording, you must tune your guitar to a tuning fork or an electronic tuner.

Second, most recordings have a variety of instruments playing simultaneously. For the beginner, it is difficult to tell right away what chord is being used. Third, the piece may be played in a strange (for the guitar) key (e.g., Bflat), so that you will need a capo (unless you have learned the barre chords). In this case, it is sometimes hard to find the key, since there are (theoretically) 12 different keys that might be used. Fourth, this article has addressed the playing of simple melodies by ear. More complex music may involve chords than the three basic harmonically related chords (e.g., minor chords), and more complex chords (e.g., diminished, augmented, ninth, eleventh, and thirteenth chords). If you want to play a complex piece, you should purchase the sheet music at a music store.

When you begin to learn to play by ear, you will practice on one or two songs, but you should quickly move on to playing scores, and then hundreds, of songs. You can get the words to many songs from a number of Internet sites, from so-called "fake" books, or you can simply copy the words down from recordings that you own. The problem with fake books is that the print is small. If you memorize the song, then that is fine. On the other hand, it takes some time to memorize songs, and this will delay your learning to accompany a large number of songs. Also, your repertoire will grow rather slowly (unless you have a photographic memory!). What I use is a three-ring binder filled with full-sized (8-1/2 by 11 inch) sheets, with the lyrics written out in a large font. Most songs fit on a single page. With the large font, it is possible to read the words as you play. The tremendous advantage of this approach is that you are accompanying scores or dozens of songs right from the start. You should eventually memorize the words to many songs, but if you want to play a large number of songs right away, or play requests, you will have to rely on printed lyrics.

It will help in the long run if you do not write the chords over the words, at least for songs involving just the I-IV-V chords. Writing the chords over the words will impede your progress, because you will not learn how to change keys easily and automatically. For complicated songs involving many different chords, it will be necessary to memorize the chord sequence in a particular key.

Perhaps it is obvious, but a major drawback of playing by ear is that, since you are not playing from printed music, you have to know the melody of the piece before you can play it. If your objective is to be able to accompany songs that you like, however, this is not a problem (since you already know the melodies).

When you are first beginning to learn to play by ear, you will find it useful to listen to recordings to learn the melody, but it is difficult to accompany at the beginning. It is easier to learn to play the piece by ear first, and then learn to accompany. As you learn, this will become easier.

H. How to Determine What Key to Use

If you wish to accompany a recording, you will use the same key as it was recorded in (and use a capo if the song is in an unfamiliar key). In general, however, you may use any key for which your vocal range can sing the song. After trying to sing the song in one key, you may quickly see that parts of the song are too high or too low for your vocal range. In this case, move the key up or down accordingly, and try again.

Some songs change keys in the middle, or have a chorus in a different key. For example, the song “My Arms Stay Open Late” by Billie Jo Spears starts out in one key, and then moves up two tones for the second verse. You may play this starting in the key of E and moving to G, or starting in G and moving to A. The choice will depend on your vocal range. Similarly, Charlie Pride’s song, “Does My Ring Hurt Your Finger” drops down two tones for one verse, and then returns to the original key. You could start in A and drop down to G, or start in G and drop down to E, depending on your vocal range. The song, “San Antonio Rose,” plays well with the verse in the key of D (chord sequence D-G-A), with the chorus, a fifth above, in the key of A (sequence A-E-A-E).

Some songs are so identified with a particular singer or instrumentalist that they sound best in a particular key. Part of this is that equivalent chord progressions do not sound exactly the same in different keys, since the chords are formed differently on the fretboard.

I. Minor Chords and Minor Keys

It is difficult for the beginner to learn to play songs in minor keys. The problem is that whereas many major-key pieces contain only three (or even just two!) chords, most minor-key songs contain a large number of chords. While it is often easy to play a simple major-key piece by ear the first time, it is usually necessary to "figure out" the chords of a piece involving minor keys, or even minor chords.

The procedure for determining which chords to play in a piece involving minor chords is the same as for determining the chords of a major-chord piece -- it just takes somewhat longer to accomplish. It is assumed that you can distinguish the sound of a minor chord from that of a major chord, so that you "know" that a piece contains minor chords.

In some cases, a piece is written in a major key, but it contains one or more minor chords. For example, the song "Mountain of Love" is written in a major key, but it contains a single minor chord, in the chorus. If the song is played in the key of A, the chorus goes "(D) Mountain of love, a (Dfm) mountain of love, (D) You should be ashamed, (D) You used to be a (Dfm) mountain of love, but (D) you just changed your (E) name."

The song, "Kaw-Liga," a long sequence in a minor chord, but it is considered a major-key song. In the key of E, the verse goes Em-B7-Em, and the chorus goes E-A-E-B7-E.

The song, "It Doesn't Matter Anymore," in the key of C has verse chord pattern C-G-C-G C and chorus chord pattern Am-D-G.

Some pieces, however, are definitely written in minor keys. "The Night They Drove Old Dixie Down" can be played in the key of Am. They key of Am is the relative minor key of the key of C -- the chords of both keys are almost identical. The chords of C are C-Dm-Em-F-G-Am-Bo. The chords of Am are Am-Bo-C-Dm-E-F-G (note the sharped seventh degree, changing Em to E). The chord sequence for the verses is Am-F-Am-F-Am-C-Am-C-Am-C-Am-G, and the chord sequence for the chorus is Am-C-Am-C-Am-Dm-F.

The song, "(Ghost) Riders in the Sky" is a minor-chord composition. In the key of Am, the chord sequence is Am-C-Am-F-Am, Am-C-Am-F-Am.

J. Bridges

Some songs include "bridges," which are musical passages that form transitions between two sections of a composition. They are similar to choruses (which may be considered "bridges" between verses), but a chorus is usually in the same key as the song, whereas a "bridge" usually isn't. An example is the song "San Antonio Rose." If the verses are played in the key of D (D-G-A-D), the chorus is in A (A-E-A-E). It is often the case, as in this one, that the bridge is in the key of the dominant chord of the main key (in this example, A being the dominant of D). (The dominant chord is a "fifth" above the key chord, or the next chord (clockwise) on the circle of fifths.)

Some bridges are very short. For example, in the song, "I Don't Hurt Anymore," in the key of A, the chords of the first two verses are A-D-A-E-A, A-D-A-E-A, and the "bridge" verse is D-A-B7-E-A. In this case, the bridge is the chord B7 E (once again, from the key (E) a fifth above the main key (A)).

The song, "Slipping Around," in C has verses C-F-G-C-F-G and chorus (bridge) G-C-D-G-G7, returning to C-F-G-C.

The piece "Yellow Bird" (or "Don't Ever Love Me") in C has verse C-G-C, bridge Am-D-G-G7, and chorus C-G7-C-G7-C-F-C-G-C-F-C-G7-C.

The piece "Don't We All Have the Right to Be Wrong Now and Then," in key of A has verse A-D-A-D-A-E, A-D-A-D-A-E, bridge B7-E-B7-E, returning to verse A-D-A-D-A-E.

"Hello Walls" in the key of A has verse A-D-A-E-D-A-E-A and bridge B7-E-B7-E.

K. More Complicated Pieces

Some songs are just plain complicated! They have many chords, and are not at all restricted to a particular key. The song, "Now and Then, There's a Fool Such as I" has the chord sequence (in the key of G) G-B7-C-G-A-A7-D-D7, G-B7-C-G-D-D7-G-G7, and chorus C-D-G-Em-A-A7-D-D7, G-B7-C-G-D-D7-G. For such a song, it is most practical simply to play it in the same key from printed lyrics with the chords indicated, rather than spend the time required to memorize the chord sequence (unless you intend to play it a lot).

The piece, "Tumbling Tumbleweeds" in the key of G has the chord sequence C-B-C-G-D-G for the verse and bridge D-G-A-D. Note that this song begins in a chord different from the key chord.

L. How to Sing and Strum at the Same Time

At this point, we have learned the sequence of chords to accompany a melody, but we are not singing the song or using “interesting” strum patterns. The next step is to learn to sing instead of hum. For the time being, do not attempt more elaborate strum patterns. Learning to sing and change chords is difficult enough with a simple, monotonous strum pattern (thumb moving down over the strings every time). When you are quite comfortable singing and playing with the simple strum pattern, then try some more elaborate patterns, such as the thumb playing alternate bass strings and the fingers plucking the treble strings.

After you have mastered singing, it is time to try the pick (or plectrum). With the pick it is possible to master a wide range of strum patterns, strum faster, and play loudly. Playing loud is unnecessary if you are playing for yourself or for a few friends, but it is important if you are playing in a large group, or out in the open, or in a large room. The alternative is electronic amplification, which is fine if you have access to it. If you have only an acoustic guitar, however, it is necessary to play loud in certain situations, and the pick does a better job than the fingers.

After a few weeks of practice, you will be able to strum automatically as you sing. For many popular songs, it suffices to use the "alternate bass" strum pattern, in either 4-4 (hit a bass string and then strum the rest of the strings, then hit a different bass string and strum the rest of the strings) or three-quarter time (hit a bass string, strum twice, hit a different bass string, strum twice).

IV. Summary and Conclusion

Since I had been taught music notation in grade school and in every music instruction course I ever attended, I assumed that you needed to know music notation to play by ear. At least, I assumed that it would help. But it does not help at all. Music notation is essentially irrelevant for playing by ear. Knowing how to read music does not hurt, but it doesn’t help. In fact, it may be a disadvantage in that if you really want just to accompany songs by playing by ear, and start out by learning to read music, you invest a large amount of effort in an activity that is of no value whatsoever – effort that you believe is helping you achieve your goal, but that is not. It may create the wrong impression that learning to play by ear is very difficult (more difficult than reading music, since you still cannot do it after you learn to read music), when in fact it is no more difficult than learning to read music – it is just a different activity. From this viewpoint, learning to read music may have denied many people the lifetime pleasure of playing by ear.

Not that music notation is not extremely important, valuable, and useful. It opens up the tremendous opportunity of being able to read the music of the centuries and reproduce it. Musical notation is as important to music as written language is to language. It is essential for a large number of instruments (band, orchestra) to play together. It is essential for a song for which you do not know the melody. But it is of no value at all in helping you accompany a song in any key with the guitar!

So there you have it – a method for learning how play the guitar (and accompany songs) by ear, along with some theory to explain why it works. I know that the preceding method works, because it is the method that I used. I am sure that it will work for most people, because I had a lot of trouble learning to play by ear. This subject is not taught in the schools or in the music books that I bought. The method recognizes that knowing how to play from written music is of no advantage in helping you learn to play by ear.

I have illustrated the method for a guitar, but it would work for other chording instruments as well (piano, organ, electric keyboard).

It took me a long time to realize what was involved in playing by ear in any key. This was a shame, because I could have been accompanying songs over thirty years ago, instead of just last month (January 2000). If you want to learn to play by ear, and to accompany popular songs in any key, you can learn how to do this in a short time – a month or so, depending on how much you practice (an hour every day is recommended). I hope that this article enables you to avoid the lost time that I encountered, and helps you learn to play by ear quickly.

Good luck!

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Gaborone, Botswana
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After I placed an initial draft of this article on the Internet, I received comments from Mr. Jos S. Groot of The Hague, Netherlands. From his comments, it is clear that others have addressed the music-theory issues that I have addressed here, in particular, the problem of determining how to divide the octave. He provided me with some interesting historical information on the work of Dutch musicians. He noted that Christiaan Huygens (1629-1695) proposed dividing the octave into 31 equal steps, and that a museum in Holland has two organs in its collection that are based on this solution and are regularly played. He read that the Swiss Euler (1707-1783) adapted Huygens findings, and that

the Dutchman Stevin (1548-1620) providing the octave into twelve equal divisions. The value b=18, n=31 of Huygens solution yields b/n = .5806, which differs by .0044 from the desired value .585. It is hence not as good a solution as b=7, n=12 (b/n = .5833, difference = .0017), or b=17, n=29 (b/n = .5862, difference = .0012). Mr. Groot also noted that he had seen an article in Physics Teacher about the solution b=7, n=12.

My thanks also to my friend, Archie Taylor, for the time he spent reviewing this article. He and I spent many delightful hours jamming, with Archie on the clarinet and me on the guitar, at his homes in Spartanburg and at Fripp Island, South Carolina.